

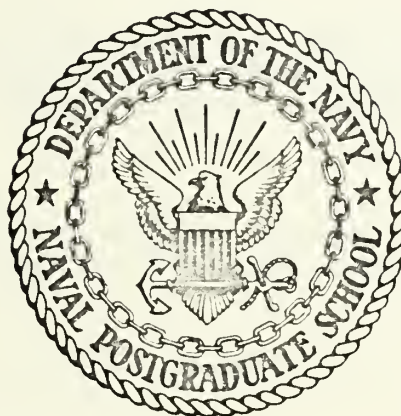
ESTIMATION OF COBB-DOUGLAS PRODUCTION  
FUNCTIONS AT A NAVAL AIR REWORK FACILITY

Charles William Bradley



# NAVAL POSTGRADUATE SCHOOL

## Monterey, California



# THESIS

ESTIMATION OF COBB-DOUGLAS PRODUCTION  
FUNCTIONS AT A NAVAL AIR REWORK FACILITY

by

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September, 1972

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FUNCTIONS AT A NAVAL AIR REWORK FACILITY

by

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## ABSTRACT

The objective of this study was to estimate Cobb-Douglas Production Functions for the aircraft rework, engine repair, and component repair programs of the Naval Air Rework Facility, Naval Air Station, North Island, San Diego, California. The production functions were estimated by multiple regression analysis from data aggregated from actual data taken from production records of the three major rework and repair programs at North Island. It was then indicated how the cost-effectiveness of a computerized work in process inventory control system could be investigated by examining trade-offs between the varying levels of the input factors of production.





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## I. INTRODUCTION

### A. ORGANIZATION AND ENVIRONMENT<sup>1</sup>

The Naval Air Rework Facility, Naval Air Station, North Island, San Diego, California, (NARFNI) is one of the seven rework facilities serving aircraft for the United States Navy and Marine Corps. It is directly responsible for all major maintenance, incorporation of technical changes, and repair of crash damage for West Coast based F-4, F-8, E-2 aircraft and West Coast based helicopters.

Fleet requirements are promulgated by Commander in Chief Pacific Fleet (CINCPACFLT). He, in promulgating these requirements, is constrained by the Department of Defense (DoD) Five Year Plan (FYDP), which in turn is determined with respect to overall national policy, and especially with respect to foreign policy, force posture deemed necessary arising from these policies, and governmental fiscal constraints. The fleet requirements dictated from CINCPACFLT represents an economic choice constrained by foreign policy commitments and national budget. CINCPACFLT has an inventory of aircraft requiring periodic maintenance, emergency maintenance, and installation of technical changes. He must keep enough aircraft operational to satisfy his

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<sup>1</sup>See Naval Air Rework Facility Instruction 7650, Cost Control Manual, 30 July 1971.



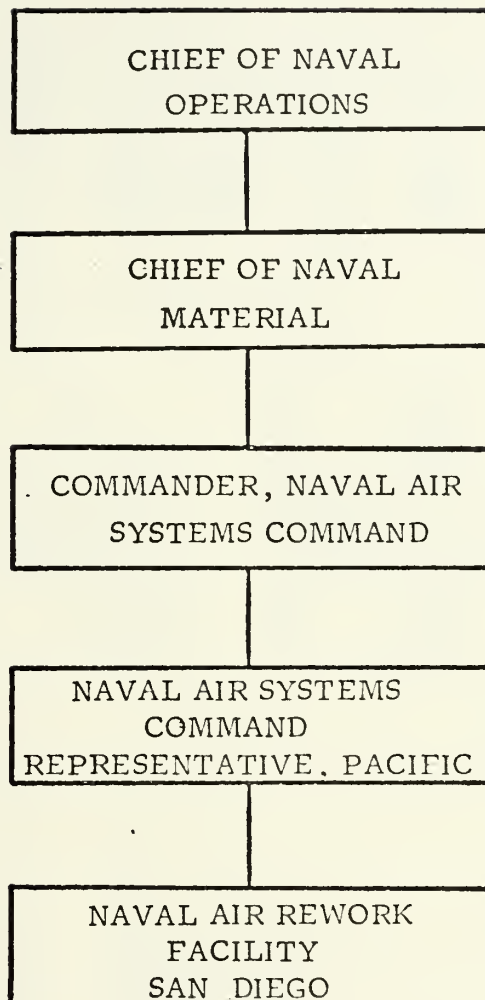
operational requirements and to support his training and readiness effort, and must somehow rotate all aircraft through rework for maintenance and change to keep the fleet hardware technically viable in the face of a supposed threat.

NARFNI is required to complete the required work load in a specific period of time at a minimum total cost to the Government. Any savings in time while accomplishing this required work load would be realized by CINCPACFLT as an opportunity to accomplish other maintenance, which otherwise would be unaccomplished, provided funds are available, or would be realized instead as an increase in aircraft operational availability, if the same work load were maintained. If the same output were produced at a lower cost, CINCPACFLT would be able to make additional economic choices, otherwise foregone, not necessarily purchasing more aircraft availability, but the items of next higher marginal value to him.

NARFNI is directly responsible to the Naval Air Systems Command Representative, Pacific, (REPPAC), (Figure 1), who through Commander Naval Air Systems Command (COMNAVAIRSYSCOM) is laterally responsible to provide aircraft maintenance to CINCPACFLT in the operational chain of command.

Organizationally NARFNI is presently composed of nine major divisions (Figure 2 and Figure 3). Each division is composed of a direct labor force and an indirect labor force, encompassing an overall labor force of approximately

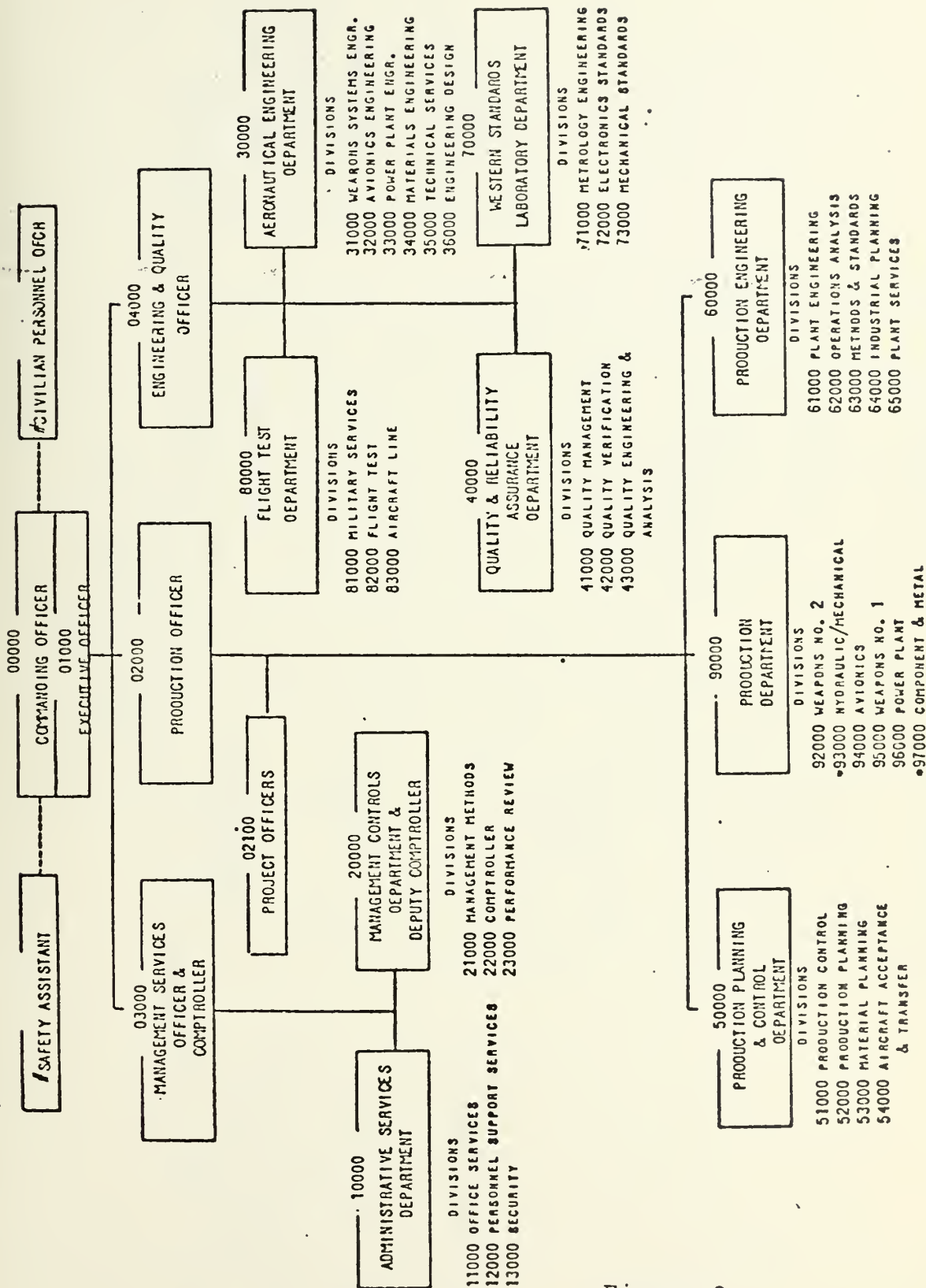




REPORTING CHAIN OF COMMAND FOR NARFNI

Figure 1



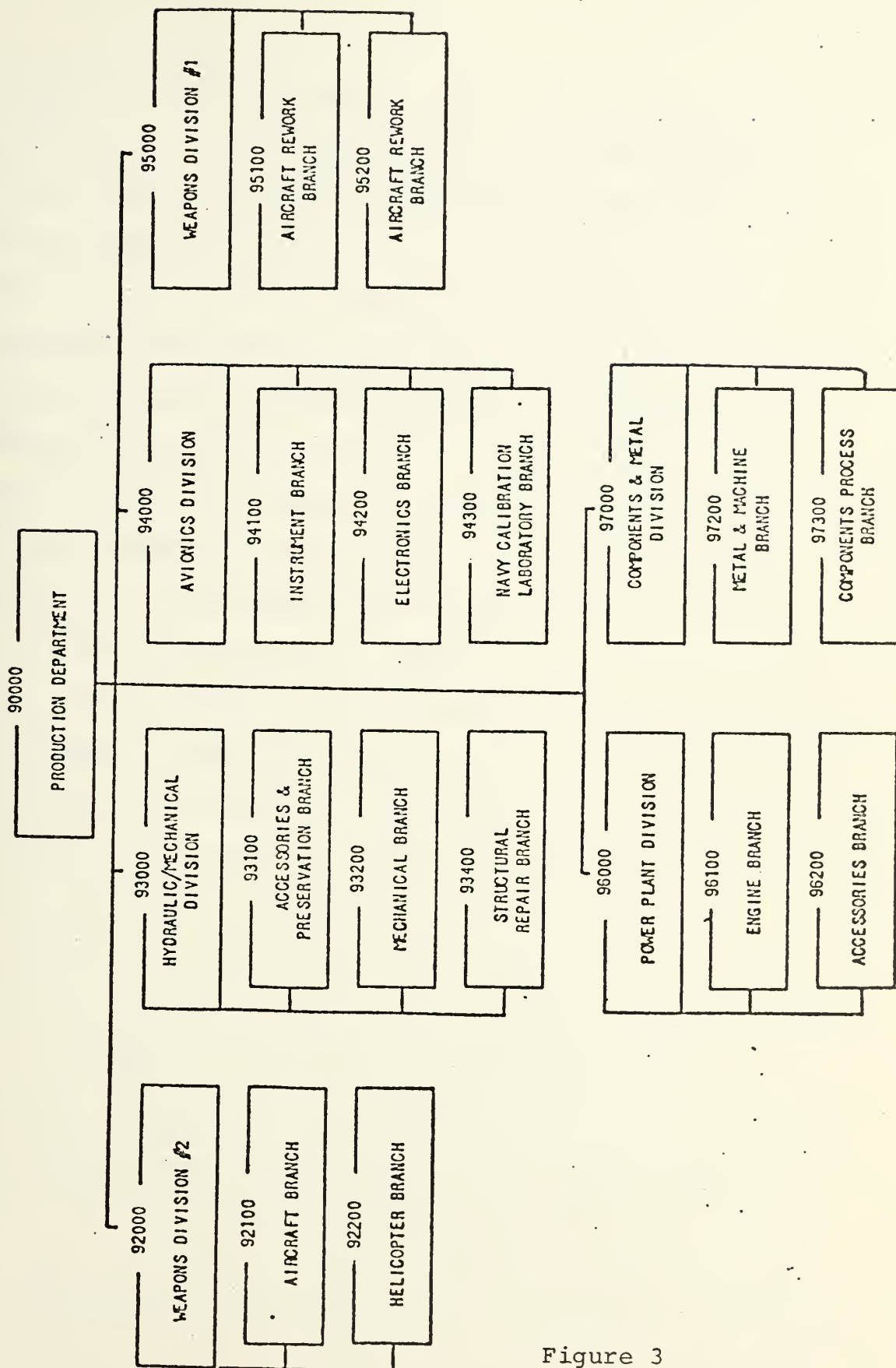


NAVAL AIR REWORK FACILITY, SAN DIEGO, ORGANIZATIONAL CHART

Figure 2







DIRECT LABOR FORCE STRUCTURE

Figure 3



6,800 personnel. The indirect labor force is made up of managerial, secretarial, supervisory, and administrative personnel; while the direct labor force is comprised of skilled tradesmen.

The physical plant of NARFNI represents a sizeable investment of Government funds in buildings, test cells, laboratories, and air force facilities. The present plant, encompassing 298 acres, is valued at \$115 million, plus another \$18 million budgeted for rebuilding and expansion of existing facilities. NARFNI has a total yearly budget of \$150 million and overhauls approximately 80 aircraft and 23,000 related components per quarter.

The highly sophisticated and complex nature of a military aircraft necessitates periodic modifications of existing systems and major overhaul of component parts. Therefore, all aircraft are scheduled for preventative maintenance on a regular Progressive Aircraft Rework Cycle (PAR Cycle) during their life span. Prior to induction into a specific PAR Cycle, an estimate, based on historical data of man hours required to update the aircraft to necessary technical standards, is obtained. The required material is ordered, and managers determine the optimal allocation of existing manpower resources necessary to accomplish the work load while retaining several manpower utilization options in the event of unforeseen work load changes. In theory, an aircraft arrives three to four days prior to scheduled maintenance,



is prepared for induction, and, on day zero, enters the system, then taking exactly the number of man hours specified by the PAR Cycle to complete overhaul.

Between quarterly planning and induction of the aircraft into the rework facility, many events may, and often do, take place that effect the predicted work load requirements at NARFNI. Extensions of deployments or immediate deployments could prevent aircraft from arriving by the scheduled induction date. Upon arrival at NARFNI, the predicted man hours required could vary due to unexpected problems, such as a lack of skilled workers, a series of crash-damaged aircraft requiring more than routine repair, or aircraft otherwise requiring more than the anticipated work. Any of these problems could conceivably double the estimated man hours required to meet the quarterly production schedule.

#### B. FUNDING AND ACCOUNTING<sup>2</sup>

NARFNI, like shipyards, ordinance plants, ammunition depots, weapons facilities, research and development laboratories, test centers, and other Navy-run activities, is funded primarily by the Navy Industrial Fund (NIF). This fund was authorized by Act of Congress in 1949; now roughly

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<sup>2</sup>See Naval Air Rework Facility Instruction 7650, Cost Control Manual, 30 July 1971.



one hundred Naval activities operate under the NIF system, with an annual volume of business of approximately \$5 billion. In essence, the NIF system provides a working capital fund for purposes specifically authorized by the 1949 law to finance continuing cycles of ongoing operations. Receipts derived from these operations are used to perpetuate the fund, thus requiring no further Congressional action. The fund consists of an initial allocation from the U. S. Treasury, accounts receivable, inventories of material, work in progress, and other current assets; subject to liabilities assumed at inception, plus those subsequently incurred in support of current operations. All work done by NARFNI is paid for initially out of the NIF which is then reimbursed from customer appropriations.

NARFNI is engaged in three major programs:

1. Progressive Aircraft Rework
2. Engine Repair
3. Component Repair

By far, most of the work is done for Naval Air Systems Command Representative Pacific, although small amounts are done for other customers, namely various Air Force and Coast Guard Commands.

In general, the cost chargeable to the customer includes direct civilian labor and most material used in the three major repair programs. If possible, the labor and material expended are identified with specific, benefitting end-products;





and, insofar as this is possible, costs charged are exact. For some of the work done at NARFNI, the cost of direct labor and materials must be prorated over groups of jobs, since the work is of bulk or of lot-size nature and cannot economically be identified with each specific end product.

In addition to the costs of direct labor and material (Figure 4 and Figure 5), an overhead cost (Figure 6) is charged to the customer to recover the costs of indirect labor, station support services and employee benefits. There are also other costs, statistical costs, not chargeable to the customer, which are carried in the accounts concurrent with the direct operating costs. These costs include some maintenance to the NARF's physical plant, the cost of government-furnished items, and certain other major components funded from appropriations outside the NIF. These statistical costs, along with the direct operating costs, provide a means of calculating total cost to the government of NARF operations.

Associated with the NIF System is the NIF Accounting System. It is a commercial accounting system in which credit and debit items are accrued and charged to the proper account in the accounting period in which they occur. These provide the basis for the budgetary decisions at NARFNI. Management is able to follow the progress of current operations and is thus able to measure performance against plan.



# CYCLE OF OPERATIONS UNDER NAVY INDUSTRIAL FUND

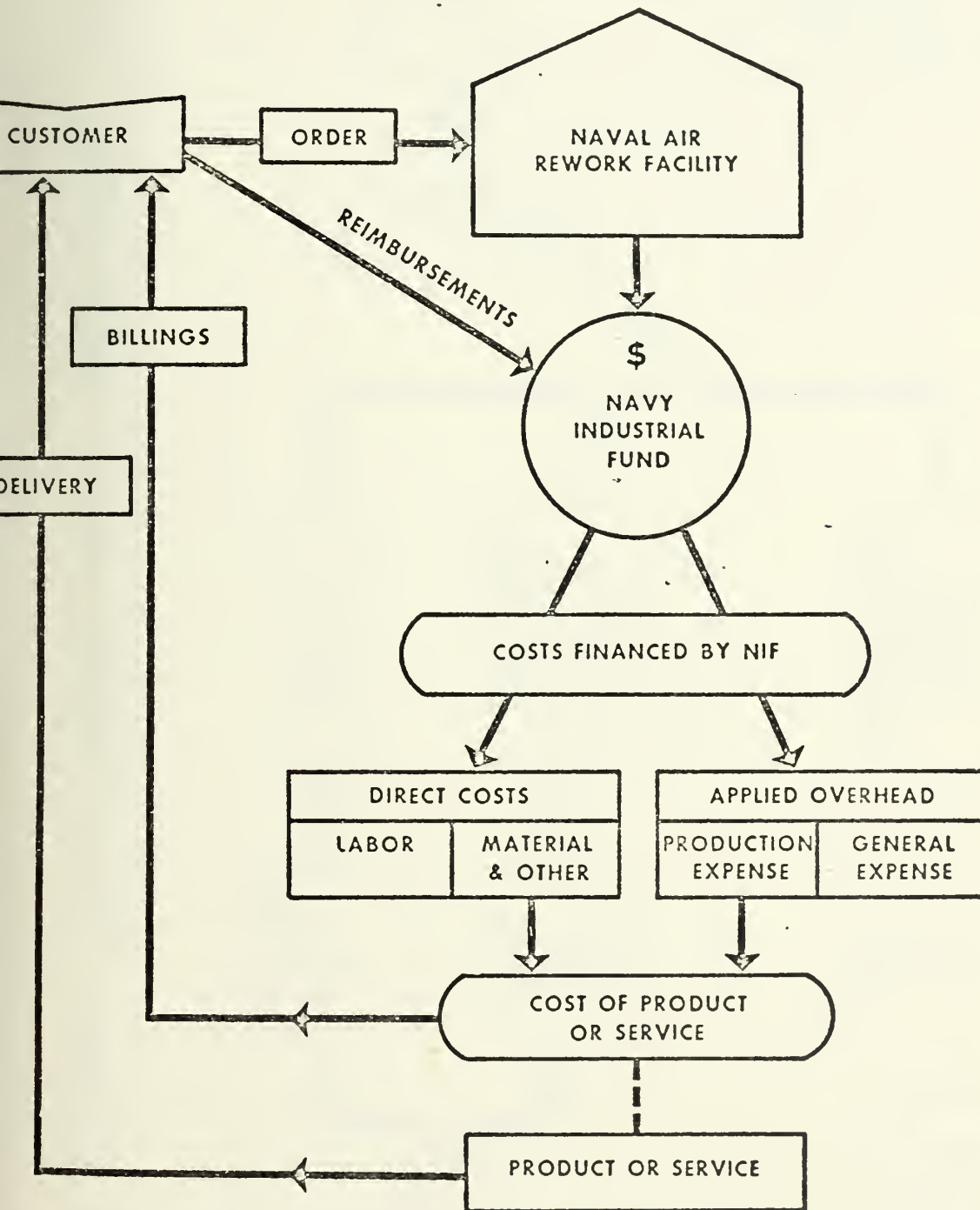


Figure 4



# JOB ORDER COSTS UNDER NAVY INDUSTRIAL FUND

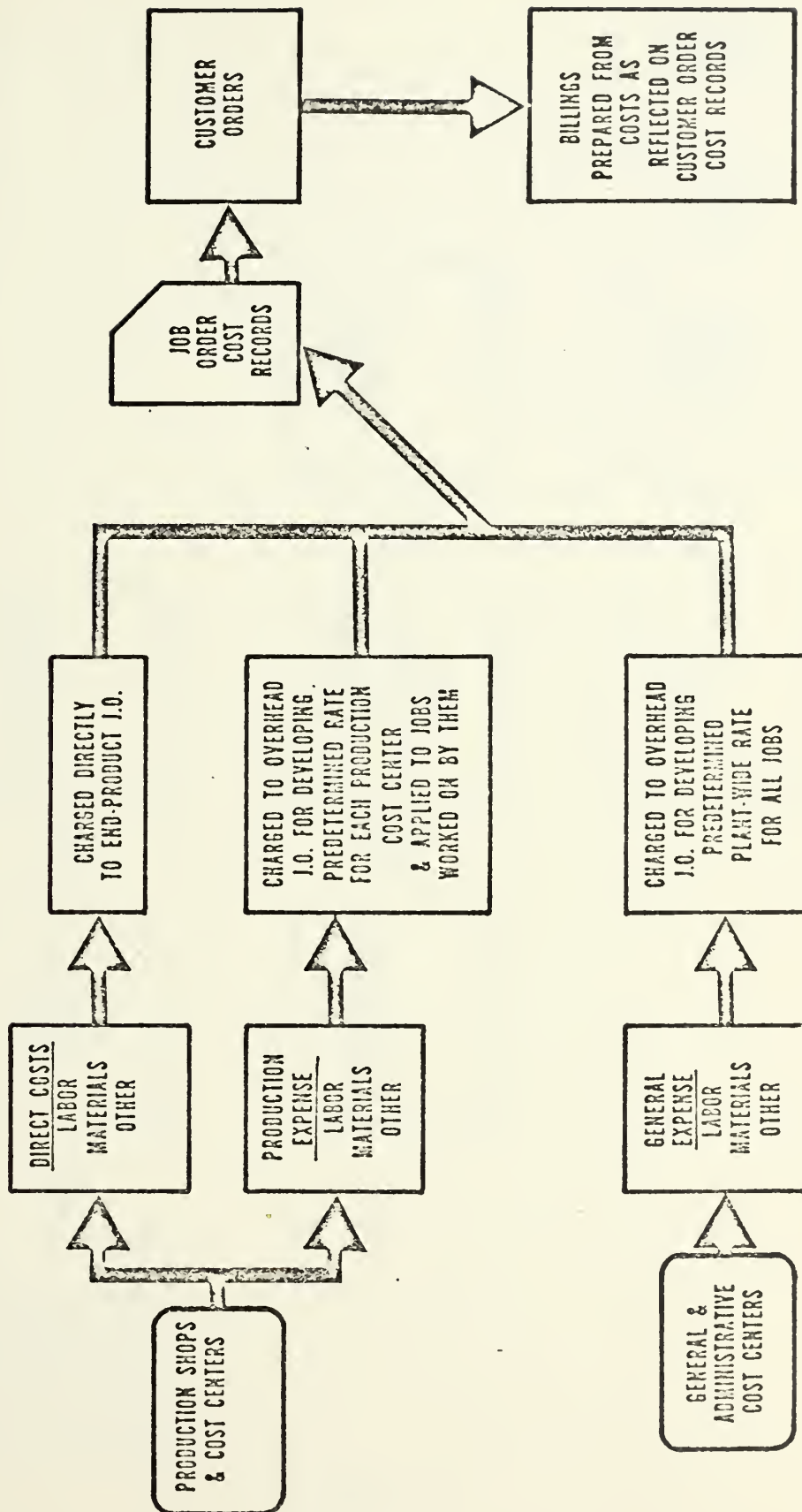


Figure 5



# APPLICATION OF OVERHEAD

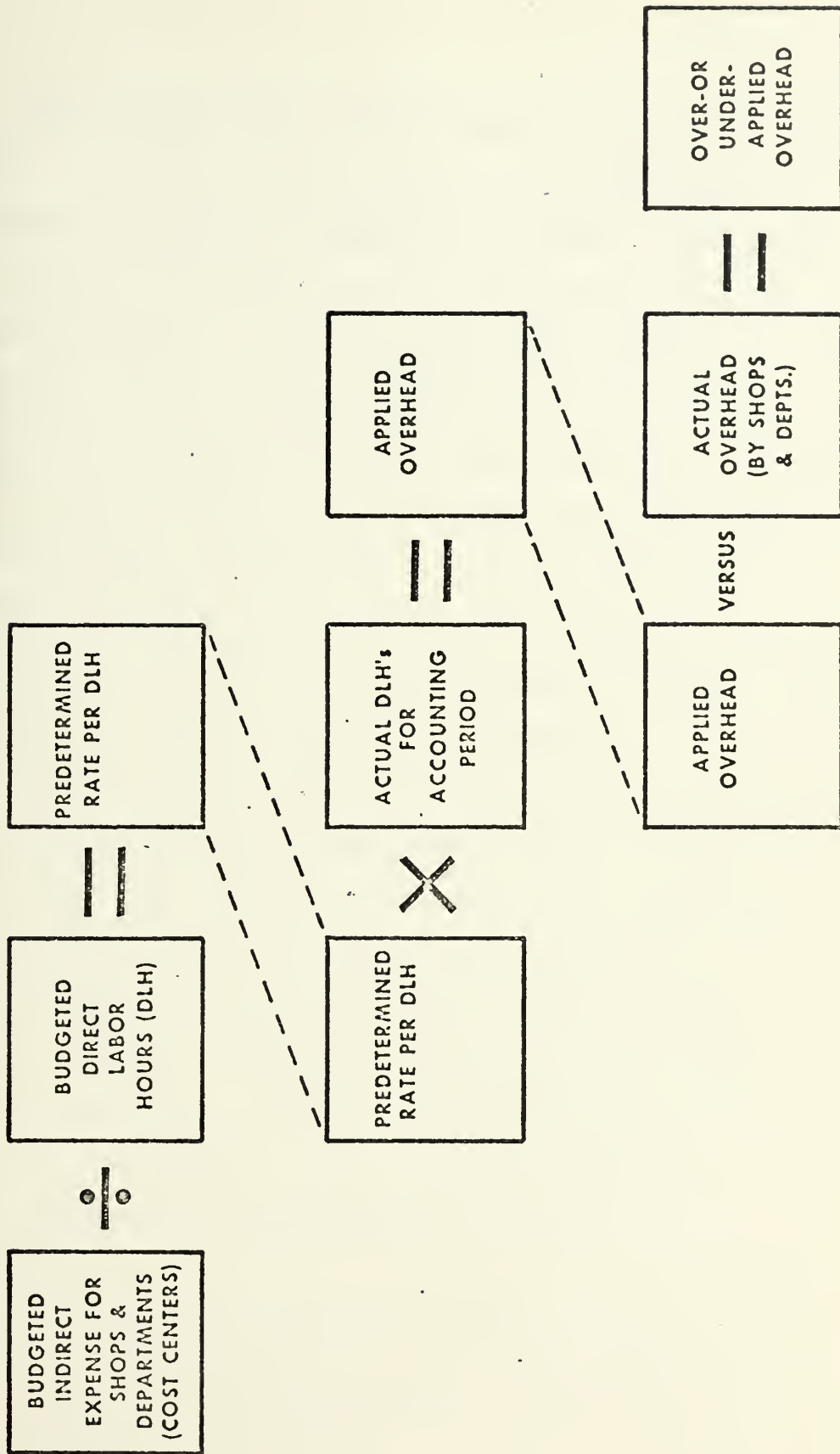


Figure 6





The Customer Order Record, NARFNI's official accounting ledger, is maintained on a weekly basis and shows the financial progress of each item in rework. Summarized in this record are entries such as the amount of money authorized by the customer, the total direct labor hours needed, the cost of labor and material actually charged to date, the amount of overhead distributed, the total amount billed to the customer to date; and also summarized is whether or not existing funds are sufficient to complete the job. This data is used by NARFNI as a basis for the negotiation of prices and rates at the Quarterly Fleet Readiness Support Conference.

NARFNI is required by COMNAVAIRSYSCOM to plan and control its financial affairs in an attempt to incur zero profit and zero loss at the end of each fiscal year. Prices for services rendered during each quarter are established through negotiations with the customer at the beginning of the quarter during the Quarterly Fleet Readiness Support Conference. NARFNI attempts to negotiate prices which balance revenues and costs, thus maintaining the NIF at a constant value. It is NARFNI's goal to balance deficits or surpluses in one quarter with compensating surpluses and deficits in the next.

Most of the work negotiated at these quarterly conferences is on a fixed-price basis. To qualify for a fixed-price contract:



The terms and conditions of the work must be specific.

NARFNI must have prior useable experience to formulate cost estimates.

The work to be accomplished must be within NARFNI's manpower and work-load ceilings.

Work done on newly introduced aircraft, on crash-damaged aircraft, and in the component program is usually not on a fixed-price basis, but is cost-reimbursable.

Performing most work on a fixed-price basis focuses attention on minimizing costs and the specific work content of each job. This promotes good cost estimation and resource allocation. The difference between actual cost and contracted price is taken up by the NIF as profit or loss. This difference influences prices and rates negotiated at the next Quarterly Fleet Readiness Support Conference. A reduction in the fund tends to raise prices in the next quarter, and the creation of surplus tends to lower prices.

A fixed-price contract can be renegotiated under the following circumstances:

If a change is developed in the technical requirements.

If a significant change is discovered in the scope of work necessary.

If the scope of the job originally requested is changed.



If a significant change occurs in the cost of material.

Either the customer or NARFNI may initiate the re-negotiation.

The Project Order serves as a contract for work done by NARFNI. When the order is accepted, it obligates funds provided from customer authorizations similar to a contract with a commercial enterprise. The Project Order indicates the following information:

The exact description of the work to be done

The completion date.

That NARFNI can reasonably be expected to complete the work in the specified time frame.

The identification of government-furnished materials.

That sufficient funds are obligated.

The accounting data for billing.

A fixed-cost or a cost-reimbursement payment.

Thus, the NIF system, along with the negotiation and contracting process, seeks to create for NARFNI an environment similar to that of industry. A contractual relationship is established which obligates NARFNI to a specifically defined task and requires that costs related to a task be accurately forecast; in most cases, a fixed price is quoted for the work. The customer is required to provide adequate funds for the work and to make an economic choice, allocating



his resources to those items he needs most. The identification of costs to a particular job allows management to monitor and control costs, to develop better pricing methods, to project realistic and efficient operating budgets, and to measure directly efficiency in terms of dollars. The NIF also provides flexibility of operations since the operating funds are free of the Congressional cycle and since management will be free to execute its own immediate control, responding in a timely manner to current plant conditions as they are interpreted.

#### C. THE WORK IN PROCESS INVENTORY CONTROL SYSTEM

In 1972 NARFNI installed a computerized Work in Process Inventory Control System (WIPICS), developed by the Rohr Corporation, Chula Vista, California (Rohr Corporation, Chapters 2-6, 1971). WIPICS is an industrial information system designed to assist in the tracking and control of inventory in the rework process. Records containing information on each item inducted into the rework process are entered into the computer system. The location, schedule and condition of each item is tracked from disassembly to reassembly by the workers in the various shops inputting data into a computer data bank by means of Touch Tone (R)<sup>3</sup> telephones.

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<sup>3</sup>Touch Tone (R) is a registered trade mark.





The WIPIC System consists of a central computer which is tied to the NARFNI shops by "touch tone" telephones and on-line printers. Inputs to the system are both on-line (direct telephone) and off-line (computer-generated tapes). Outputs are telephone audio response and on-line printer reports (direct), and computer-generated reports off-line (indirect).

The "touch tone" telephones are located in the NARF Shops and are tied to the computer through concentrators and an audio response unit. The telephone has a keyboard similar to a small adding machine. Depression of a key produces a tone which is interpreted as the value of the depressed key. The concentrator is similar to a switchboard, connecting telephones to the audio response unit which interprets a series of tonal input and responds in a prerecorded human voice. The words of the voice are contained on a spinning magnetic drum and are read off in a manner similar to that of a tape recorder.

The off-line inputs to the system, which consist of records of items going into rework, are generated by a NARF computer and placed on a computer tape. The tape is delivered daily to the WIPICS computer system and is entered in the data bank prior to the start of that day's work.

Each item entering rework is disassembled into identifiable assemblies and subassemblies. For each of these components, a computer record of information is generated. The records are placed in computer storage, retrieved and



modified as the operators input changes, and returned to storage. The assemblies and subassemblies are arranged in a hierarchy according to level of disassembly. Each record, upon inquiry, provides the operator with information of the next higher assembly, the other assemblies of the same level and assemblies of lower levels.

Induction of work begins with the generation of the original record when the off-line input tape is delivered to the WIPICS computer. When work actually begins, the operators in the shops update this record by entering transactions by means of the "touch tone" telephones. These transactions include information such as identification of an assembly and its associated hierarchial assembly chain, receipt or movement of a job or assembly, completion status, delay and reason for delay, or how urgently the part is needed. At any stage of rework, if disassembly or reassembly occurs, the assembly hierarchy is updated.

Input of data requires little training on the part of the shop personnel. Records are updated in a sequential audio question-and-answer format by means of the "touch tone" telephone and the interaction with the audio response unit. The computer verbally prompts the worker at each step of the process, asking for the correct item of information in proper sequence. If at any time the operator enters information out of sequence, or if the information entered is invalid, the computer again verbally informs him of his error.



A shop manager can interrogate the information system to get desired data on specific items which he sees as critical to his production schedule. It is important to note that he gets only the information he desires and does not have to search through long lists of computer output for the relevant item. The hierarchy of assemblies and subassemblies enables him to observe the impact of his production schedule on the work center to which he must input components, and to observe the impact of the production schedules of those work centers which input components to him.

This is especially useful in reassembly work centers. The manager can iterate the system to find the status of assembly of the components which he must assemble in his shop. Considering the large physical size of the NARF plant, this easy retrieval of specific information is quite a boon when it is realized that the alternatives are searching through volumes of computer output for a few specific items or else "sending someone to find out."

Sets of off-line reports are routinely generated by the WIPICS computer during periods of WIPICS inactivity, usually nights and weekends. Such sets consist of any reoccurring data that management may select to be displayed.

NARFNI has the only WIPICS system presently in operation in the Navy. Although the system is expensive, it may be true that through its use, greater production, either in more output per unit time, or in the same output requiring fewer



resources, is realized. Thus, WIPICS may be shown to be cost-effective and appropriate for installation at the other six NARFs.

#### D. SCOPE OF THIS STUDY

It is the purpose of this study to investigate a methodology of relating resources consumed to a measure of production; so that trade-offs between resources, namely material and labor, and time in repair, can be observed. Specifically, the methodology will be to formulate a Cobb-Douglas production function through regression analysis of actual production data taken at NARFNI from March, 1969, until August, 1971.





## II. THEORETICAL BACKGROUND

### A. COBB-DOUGLAS PRODUCTION FUNCTION THEORY

If there are two factors of production, a production function in the simplest Cobb-Douglas (Nerlove, 1965, and Walters, 1963) form is:

$$Y = A X_1^\alpha X_2^\beta \quad (1)$$

$$\begin{aligned} Y, X_1, X_2 &> 0 \\ \alpha, \beta &\geq 0 \end{aligned}$$

Properties of this production function are:  $\alpha$  and  $\beta$  are elasticities of production with respect to  $X_1$  and  $X_2$  respectively.

The function is homogenous of degree  $\alpha + \beta$ . If  $\alpha + \beta$  exceeds unity, there are increasing returns to scale:  $\alpha + \beta$  equal to 1 indicates constant returns to scale:  $\alpha + \beta$  less than 1 indicates decreasing returns to scale.

Marginal physical productivity of one variable,  $X_1$  for instance, declines if  $\alpha < 1$  and  $X_1$  input is increased.

The marginal rate of substitution is  $\alpha X_2 / \beta X_1$  and the elasticity of substitution is unity.

Natural logarithms of both sides of Equation (1) can be taken resulting:



$$\log Y = \log A + \alpha \log X_1 + \beta \log X_2 \quad (2)$$

Equation (2) is now in a form permitting estimation of the coefficients,  $\alpha$  and  $\beta$ , by linear regression.  $\log Y$  is the dependent variable;  $\log A$  is a constant term;  $\log X_1$  and  $\log X_2$  are the independent variables, and  $\alpha$  and  $\beta$  are the coefficients to be estimated by  $\hat{\alpha}$  and  $\hat{\beta}$ . It is possible after estimation of coefficients to return to form of equation (1) with the variables in terms of the original units instead of logarithms by replacing  $\alpha$  and  $\beta$  with  $\hat{\alpha}$  and  $\hat{\beta}$  and taking antilogarithms of both sides of the equation. The result is:

$$Y = A X_1^{\hat{\alpha}} X_2^{\hat{\beta}} \quad (3)$$

Using equation (3), it is possible to observe the impact of changes in the factor of production on output. If a basic change occurs in the production process, the parameter estimates,  $\hat{\alpha}$  and  $\hat{\beta}$ , will change, and one can observe the gains or losses in production by holding the inputs,  $X_1$  and  $X_2$ , constant and applying the estimates,  $\hat{\alpha}$  and  $\hat{\beta}$ , obtained from data taken before the change in the production process to equation (3). The same can then be done with  $\hat{\alpha}$  and  $\hat{\beta}$  obtained from data taken after the change in production, and the two calculated outputs can then be compared.

If physical units are used as the measure of the variable factors of productions, and if many different factors are



combined in the production, the estimation of a production function can be quite difficult. One may estimate a production function using the procedures outlined above, estimating a coefficient associated with each different variable in the production process. This becomes an enormous problem in a large production process with many different factors, and possibly many different products. Usually data is not available in sufficient detail to permit this procedure. It is then necessary to aggregate the data to reduce the number of variables to a workable set. It may be possible to aggregate over similar physical units; that is, simply to treat several of the variables as one variable, and for each separate observation, sum the similar variables together. The number of variable factors of production in the production function would then be reduced to the number of groups of similar variables. If more than one product is produced, the products can be sorted into similar groups, and the corresponding groups of factors for each group of products, summed. This will reduce the number of production functions to be estimated in a multi-product process.

Often it is necessary or desirable to describe a multi-product process with just one production function; or it may be desirable to simplify a production function from many variable factors of production to just a few. If all similar physical factors have been aggregated into groups, often a



transformation is performed on some or all of the variables to produce surrogate factors which may permit further aggregation. The most common variable of this kind is dollar value of the factors of production, standing for the physical units of the factor. Using dollar value as a surrogate is especially convenient, since dollar costs are normally recorded carefully for factors of production in a commercial enterprise; but strict accounting for physical factors as they are transformed into finished products is rare.

Dollar costs of physical variables, standing for physical variables, will accurately describe the production process only if the manager allocates his resources, both the physical and financial, at the same level of efficiency as time passes. He will be inefficient to the degree that he purchases some factors which he cannot, or does not, use to their full extent in the production process. The descriptive power of costs rather than physical units as explanatory variables is reduced if the manager's efficiency varies over time in the allocation of his resources. Often it is not clear just how efficiently resources are allocated among opportunities over time. If dollar costs are used as surrogate variables for physical variables, it is a tacit assumption that the manager's efficiency in resource allocation is constant.

It is also an assumption that the dollar cost per unit of each resource is constant over time. If the time period





covered by the data base is rather short, this is usually a good assumption; but if the time period is long, or if a static analysis is attempted comparing two estimated production functions based on two widely separated time periods, prices should be inflated or deflated by a proper index with respect to some base price, at some chosen time (Spurr and Bonini, pp. 427-457, 1967).

It is not necessary that all factors of production be either all physical or all surrogate; the production function may contain variable factors of both, or may contain factors of only one type.

It is important to remember that no special favorable connotation is to be placed on the term production. Often the product of a process is quite negative in connotation. The product of an efficient wrecking crew is many demolished buildings. Also, there may be several variables in a production function which, while taking the same form in the production function as a physical or surrogate variable, does not measure an actual input to production, but rather a symptom of an environmental condition. For example, the number of items in the production process at any one time may be a good measure of shop congestion which may have a distinct impact on production. These environmental conditions may tend to augment or diminish production. The coefficients associated with these terms in the production



function, however, are estimated in the same manner as those associated with physical or surrogate variables.

## B. REGRESSION ESTIMATION TECHNIQUE

Suppose that:

There are  $n$  observations on  $m$  variables taken while observing a production process. Each observation consists of the recording of the  $m$  random variables  $X_i$  ( $i = 1, \dots, m$ ) which are known as the independent or explanatory variables, and the recording of another random variable,  $Y$ , which is dependent on the above independent variables.

The  $n$  observations on  $Y$  can be arranged in an  $(n \times 1)$  column vector and the  $n$  observations on the  $X$  variables can be arranged in an  $(n \times m)$  matrix.

$$Y = \begin{pmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_n \end{pmatrix}$$

$$X = \begin{pmatrix} X_{11} & X_{12} & \cdot & \cdot & \cdot & X_{1m} \\ X_{21} & X_{22} & & & & X_{2m} \\ \cdot & \cdot & & & & \cdot \\ \cdot & \cdot & & & & \cdot \\ \cdot & \cdot & & & & \cdot \\ X_{n1} & X_{n2} & \cdot & \cdot & \cdot & X_{nm} \end{pmatrix}$$



X is related to Y by the relationship

$$Y = X\beta + \epsilon$$

where

$$\beta = \begin{pmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_m \end{pmatrix}$$

is an  $(m \times 1)$  vector of constants.

Let  $b$  be an  $(m \times 1)$  vector of coefficients, each of whose element  $b_i$  is an estimate for the corresponding  $\beta_i$

Also

$$e = Y - Xb$$

is an  $(n \times 1)$  vector which is equal to the discrepancy or difference between the actual observed value of Y and that predicted by the relationship

$$\hat{Y} = Xb$$

where  $\hat{Y}$  is an estimate for Y

$E(\epsilon)$  for fixed values of  $X_1 \dots X_m$  is zero.



The disturbance terms,  $\epsilon_i$ , ( $i = 1 \dots n$ ), are independent of each other.

The values of the residuals are normally distributed with zero mean and constant variance  $\sigma^2$ .

$$b = (X'X)^{-1} X'Y \quad (1)$$

is the maximum variance unbiased estimator of  $\beta$ , and can be calculated from  $X$  and  $Y$ .

The relationship:

$$Y = Xb + e$$

is known as the regression equation and minimizes the sum of squares of the discrepancies of the observations from the regression plane. The vector  $Xb$  represents the part of  $Y$  which is described by the independent variables and can be decomposed into these variables.

The residual  $e$  represents the part of  $Y$  which cannot be explained by the independent variables. (Theil, pp. 35-37, 1971).

Thus

$$\hat{Y} = Xb$$

is a least-square estimator for  $Y$  and is in error by the difference

$$Y - \hat{Y} = Y - Xb = e$$

The variance of an estimator is called its sampling variance, but usually we cannot know the true variance parameter of the true distribution of the estimator. The variance  $\sigma^2$  can, however, be replaced by its unbiased estimator





$$s^2 = \frac{1}{n-1} \sum_{i=1}^n (Y_i - \bar{Y})^2 \quad (2)$$

$$s^2 = \frac{1}{n-1} (Y - Xb)'(Y - Xb)$$

and

$$s = \sqrt{s^2}$$

is an estimate of the standard deviation.

Since  $Y$  is distributed normally, a confidence interval for the estimators can be calculated using a  $t$ -statistic. Thus, a quantitative method for judging the quality of our estimates of  $Y$  is available. A confidence interval for each of the estimated regression coefficients can be computed in a similar manner, since equation (1) shows that the  $b$  vector is a linear combination of the observation on  $Y$ , and  $Y$  as seen above is distributed Normal  $(\mu_Y, \sigma^2)$ .

$$\frac{Y - \hat{Y}}{[(n-1) s^2]^{1/2}} (n-1)^{1/2} = \frac{Y - \hat{Y}}{s}$$

The above ratio is distributed  $t(n-1)$ .

Using tables for the  $t$  distribution, the limits,

$$\pm t_{\alpha}(n-1)$$

where:

$1 - \alpha$  = desired confidence level can be found from



$$P \left[ -t_{\alpha/2}^{(n-1)} \leq t^{(n-1)} \leq t_{\alpha/2}^{(n-1)} \right] = 1 - \alpha$$

In time series regressions, there are often several variables which will be highly correlated. The likelihood of this situation increases as the number of explanatory variables increases. When two variables or several variables are jointly highly correlated, a near-linear relationship exists and the standard errors of the estimators of the coefficients in the regression equation increase.

Suppose a near-linear relationship exists between observations on two dependent variables.

Then:

$$X_{ik} = aX_{jk} - v_k$$

$i \neq j$  are independent variable indexes

$k$  = the observation index

$a$  = any constant

$v$  = any small constant

( $v = 0 \Rightarrow$  exact linearity)

so for the two-variable case:

$$X'X = \begin{pmatrix} \sum X_i^2 & a \sum X_i^2 + \sum X_{i,v} \\ a \sum X_i^2 + \sum X_{i,v} & a \sum X_i^2 + \sum v^2 + 2a \sum X_{i,v} \end{pmatrix}$$



thus:

$$|X'X| = \sum X_i^2 \sum v^2 - (\sum X_i v)^2$$

therefore:

$$\lim_{v \rightarrow 0} |X'X| = 0$$

$$(X'X)^{-1} = \frac{\text{adj}(X'X)}{|X'X|} \quad (5)$$

as  $v \rightarrow 0$

$$|X'X| \rightarrow 0$$

then from equation (5)

$$(X'X)^{-1} \rightarrow \infty$$

since

$$\text{Var } b = \hat{\sigma}^2 (X'X)^{-1}$$

$\hat{\sigma}^2$  is estimated from the data

$$\lim (\text{Var } b) = \infty$$

$v \rightarrow 0$



Therefore, if a near-linear relationship (v close to zero) exists, then the estimates for the b vector have very large variances and thus large standard errors. This condition is known as collinearity (Theil, pp. 147-154, 1971) and leads to the acceptance of the hypothesis:

$$b = 0$$

Therefore, it is not possible to obtain accurate estimates of the regression coefficients when two independent variables participate in a linear or near-linear relationship over the observations in the data base. It may be possible, however, to simply omit one of the variables in the linearly dependent set, obtaining a linearly independent set of explanatory variables and then to proceed with the estimation of the regression coefficients as given by equation (3).

Recall that the assumed relationship

$$Y = Xb + e \quad (6)$$

explains the dependent variable, Y, in terms of the independent variable, X, and a random, or unexplained residual disturbance,  $\epsilon$ .

Premultiplying each side of equation (6) by its own transpose gives:

$$Y'Y = (Xb - \epsilon)' (Xb - \epsilon)$$

$$Y'Y = b' X' Xb + \epsilon'\epsilon + 2b'X'\epsilon$$

$$Y'Y = b'X'Xb + \epsilon'\epsilon$$





which can be interpreted as a measure for the degree of variation of the dependent variable,  $Y$ , and is in fact the sum of the squares of its elements.

Let  $R^2$  equal the fraction of this sum of squares which can be explained by the independent variables, and  $1-R^2$  be the fraction of the sum of squares which can be explained by the residuals.

$$R^2 = \frac{b'X'Xb}{Y'Y}$$

and

$$1-R^2 = \frac{\epsilon'\epsilon}{Y'Y}$$

The coefficient  $R$  (non-negative square root of  $R^2$ ) is known as the multiple correlation coefficient (Theil, p. 164, 1971) associated with the regression given in equation (6).  $R^2$  is known as the coefficient of determination. The closer  $R$  and  $R^2$  are to unity, the better the performance of the explanatory independent variables in explaining the dependent variable. It is impossible, however, to use  $R$  as an exact measure for how well the independent variables explain the dependent variable. Generally, the more the data base is aggregated, averaged or prorated, the higher the values of  $R$  will be; since all aggregation processes on a random variable tend to attenuate the effect of rare but extreme values of the random variable on the aggregated measure. The apparent variance in the process the random variable measures is thus reduced.



### III. DATA AND VARIABLES IN THE PRODUCTION PROCESS

#### A. THE DATA BASE

Production records for all work in the aircraft rework, engine repair, and component repair programs were available as a data base for the estimation of production functions for NARFNI. For the aircraft rework and engine repair, the following data was available for each item inducted into the production plant at NARFNI.

Type of unit inducted

Job number

Type of work to be accomplished

Induction date

Production date

Workload norm (a negotiated estimate of number of man hours needed to complete a job)

Actual man hours expended on air frame changes  
(changes updating the air frame, not corrective maintenance)

Actual direct man hours expended exclusive of man hours expended on air frame changes

The dollar cost to NARFNI for the direct labor expended on a specific job

The dollar value of the overhead distributed to a specific job number

The NIF rate (Figure 7)



# COMPOSITION OF NIF RATE

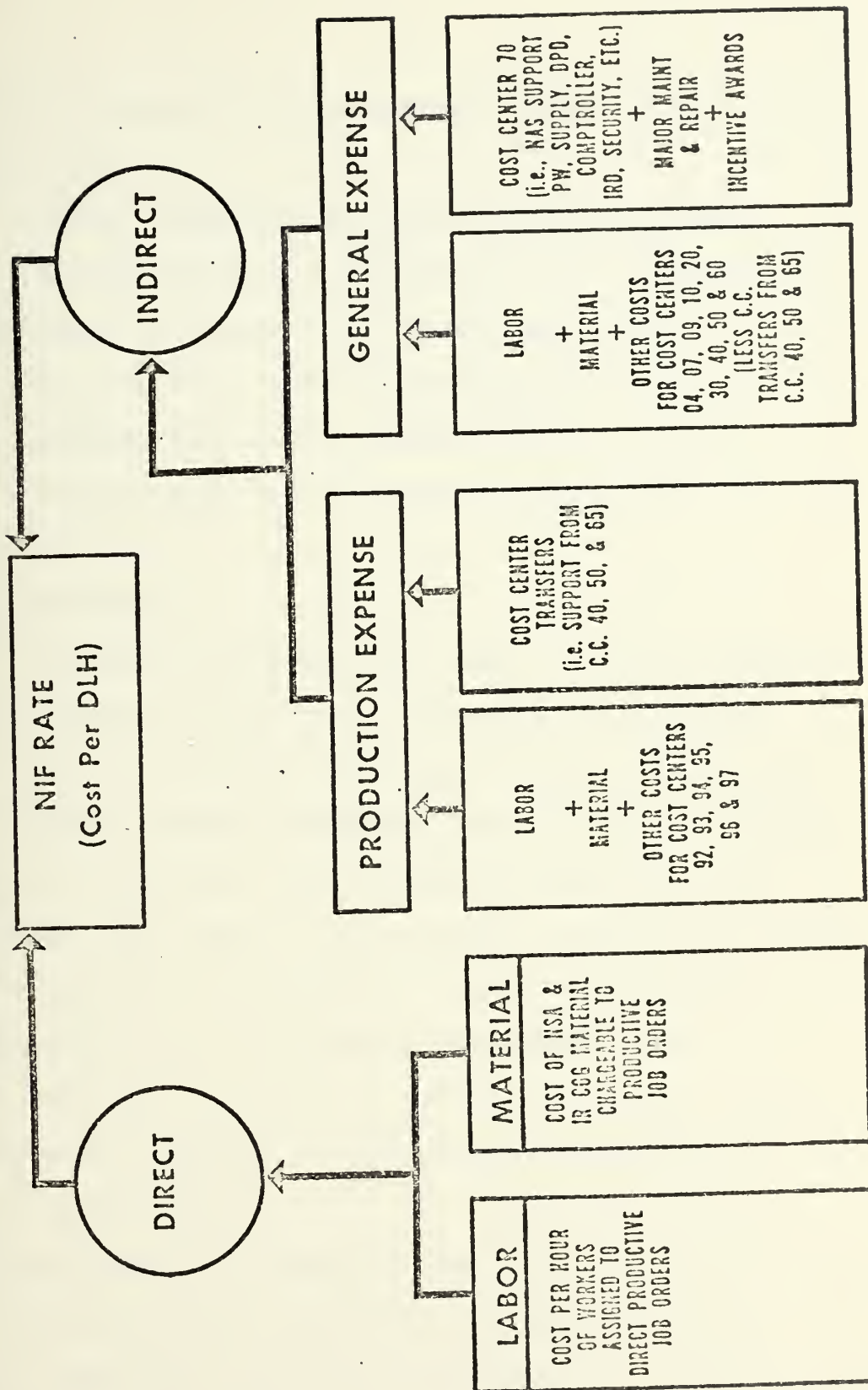


Figure 7



For the component repair program, the following data was available:

Number Components completed during the month

Average Workload Standard per Unit (the average estimated hours required to complete a job)

Average Unit Direct Hours Expended

Average Unit Direct Material Cost

Average Unit Overhead Costs Applied

NIF Rate

Since NARFNI:

Performs maintenance and repair on many different types of aircraft engines and components of differing complexity and requiring different degrees of maintenance.

Requires many different technical skills, different levels of proficiency, and different materials in the maintenance and repair of the aircraft, engines and components; (each of the above is actually a separate resource which must be accounted for in the production process).

Provides no accurate records showing how the physical materials are combined to produce each operational aircraft, engine, or component.

It was decided to aggregate the available data for aircraft and engines by a proration and grouping scheme. The data for components was already aggregated on a monthly basis.

Recall, production load norm is an estimate of the number of man hours required for NARFNI to perform a specific





package of work. This represents a budget-constrained economic choice on the part of REPPAC, in that REPPAC purchases only the maintenance for which funds are authorized by higher authority. Funds are available to REPPAC to the extent that the need for operational aircraft is dictated by the nation's foreign, defense, and fiscal policies. Therefore, production load norm was taken to be a reasonable measure of value to REPPAC, the customer, of the unit of maintenance or repair. REPPAC desires to minimize the cost of maintenance, but not to the extent of hampering the efficiency of one of its maintenance sources, NARFNI. NARFNI naturally desires to maximize the unit price of maintenance, to comfortably be able to perform its assigned work load and to maintain the NIF at a constant level from quarter to quarter. This conflict is resolved in the Quarterly Fleet Readiness Support Conference where prices and workloads are negotiated between NARFNI and REPPAC.

Norm data was available for individual jobs, which taken altogether, make up groups of similar products. Within a specific group, the work was similar and the norms were clustered quite closely. The work content varied greatly among groups and the group average norms varied widely. Therefore, since one production function was desired for each of NARFNI's major maintenance programs, aircraft, engines and components, it was necessary to aggregate the



norm data over both the aircraft and engine programs which encompassed many diverse products with widely varying norms. The component data was already aggregated. Since it was assumed that the last hour of norm of each physical product purchased by REPPAC was equal in utility, not dollar cost, to the customer, the product could then be considered to be uniform in units of hours of norm, not a multi-product process as the various physical outputs would suggest. It was the plan to relate explanatory variables obtained from the available data base to the measure of effectiveness, hours of norm, which was considered the dependent variable in a regression equation.

Since the dependent variable, hours of norm, was aggregated over NARFNI's aircraft and engine programs, the independent variables were required to be aggregated similarly.

Physical units of labor consumed, man hours, were recorded for each individual job. Man hours, themselves, were aggregated units in that they did not reflect explicitly the different skills applied to the production process, or the different levels of proficiency which existed within the separate skills. Actually one man hour from a master jet engine mechanic is not the same as one man hour from an apprentice jet engine mechanic; nor is it the same as one man hour from a master metalsmith. Also, the value of each man hour was not reflected



in the man hour data. Wage scales for different skills and levels of proficiency vary, and the cost of an overtime man hour is more than that of a man hour expended during normal working hours. Total direct labor costs for a benefitting end product were available, so the average cost of a man hour for any job could be obtained. Nothing could be said, however, of the combination of skills needed, the amount of overtime utilized or how the density of work varied during the time a job was in NARF. It was considered that both total man hours expended and some consideration of the value of man hours expended could have separate effects on production. Since it was impossible to trace allocation of labor by type, degree of skill, density, and value to each individual end product, it was decided to prorate direct man hours and direct labor costs uniformly over the time each item spends in NARFNI.

#### B. DATA AGGREGATION<sup>4</sup>

Assume that a job is inducted into NARFNI and  $t$  days later is finished (produced). During those  $t$  days, assume that  $L$  direct man hours were expended for a direct cost of  $D$  dollars. Prorated man hours expended ( $PL$ ) and prorated direct labor costs ( $PD$ ) are given by

---

<sup>4</sup>For the Raw Data, See Spooner, Appendix A, 1972.



$$PL = \frac{L}{t} \text{ and } PD = \frac{D}{t} \quad (1)$$

These prorated measures, PL and PD, may be aggregated over all jobs in the NARF by summing for each day the daily prorated values of all the jobs in the shop that day. Thus, a day-by-day time series is generated. The daily aggregated values, APL and APD, are given by

$$APL_j = \sum_{j=1}^T \left[ (z_j) \sum_{i=1}^n (PL_i) \right]$$

and

$$APD_j = \sum_{j=1}^T \left[ (z_j) \sum_{i=1}^n (PD_i) \right] \quad (2)$$

where

$$z_j = \begin{cases} 1 & \text{job in NARFNI on day } j \\ 0 & \text{job not in NARFNI on day } j \end{cases}$$

i = index of jobs in data base

j = index of days in data base

T = total number of days covered by data base

n = total number of jobs in data base

so that

$$APL_1, APL_2, \dots, APL_T \quad (t = 1, \dots, T) \quad (3)$$

and

$$APD_1, APD_2, \dots, APD_T \quad (t = 1, \dots, T) \quad (4)$$





are time series for the daily aggregated data, APL and APD, and give the total number of prorated direct man hours and prorated direct labor dollars expended on each day in the data base.

No detailed information on physical material usage was available at NARFNI. Data on material costs assigned to each separate job was available; however, conversations with NARFNI personnel indicated several inaccuracies that the data base might contain.

First, material which is used in large lots, or is used in bulk quantity, must necessarily be prorated over jobs or groups of jobs since it is economically impossible to assign such material costs job by job.

Second, the physical production date differs from the financial production date. The maintenance or repair of the physical entity is generally completed and the aircraft engine or component returned to an operational fleet unit before NARFNI receives billing for all material used in the repair. The customer, REPPAC, is billed at the financial closing date. Often billing for material is received at NARFNI after the customer is billed for the work. In these cases, the material costs must be assigned to some job other than the actual benefitting job. In spite of these difficulties, these material cost data were the only material cost data available; and since conversations with NARFNI personnel indicated that



the magnitudes of these inaccuracies were probably small compared to the average material costs of most of the jobs at NARFNI, it was considered that these data must be used and would provide a fairly accurate description of the impact of material costs on production.

Material costs were prorated in the same manner as direct labor costs in equation (1). Again, it was assumed that a job was in the plant at NARFNI for  $t$  days, and  $M$  dollars were spent on direct material costs, as shown by NARFNI records. The prorated material costs were:

$$PM = \frac{M}{t} \quad (6)$$

The aggregate prorated material costs (APM) were computed in the manner of equation (2) with  $Z$  as given by equation (3).

$$APM_j = \sum_{j=1}^T \left[ (Z_j) \sum_{i=1}^n (PM_i) \right] \quad (7)$$

$i$  = index of jobs in the data base

$j$  = index of days in the data base

$T$  = total number of days covered by the data base

$n$  = total number of jobs in the data base

thus

$$APM_1, APM_2, \dots, APM_T \quad (8)$$



is a time series for the daily aggregated data, APM, and gives the total number of prorated direct material dollars expended throughout the entire NARF on any day.

It was proper to give some consideration to production rate and to environmental conditions such as shop congestion. Conversations with NARFNI personnel indicated that it was their opinion that induction rates were fairly constant, independent of time, while production rates were low at the beginning of a quarter and high at the end of a quarter. The average daily induction and production rates were computed, using a moving block average. (Spurr and Bonini, pp. 512-516, 1967). These rates were computed from both the aircraft and engine programs in terms of hours of norm arriving or leaving per day, actual man hours of work arriving or leaving per day, and actual physical units arriving or leaving per day. (Figure 8 and Figure 9). The results were not sensitive to changes in the size of the moving window in the range of seven to twenty-one days. These plots tended to confirm NARFNI personnel in their beliefs about the production and induction rates over time. Production did increase toward the end of a quarter and did drop off sharply at the beginning of a quarter. The induction rate did have some variation over time, but it was about one-fifth as great as the variation of the production rate.



# AIRCRAFT INDUCTION AND PRODUCTION RATES

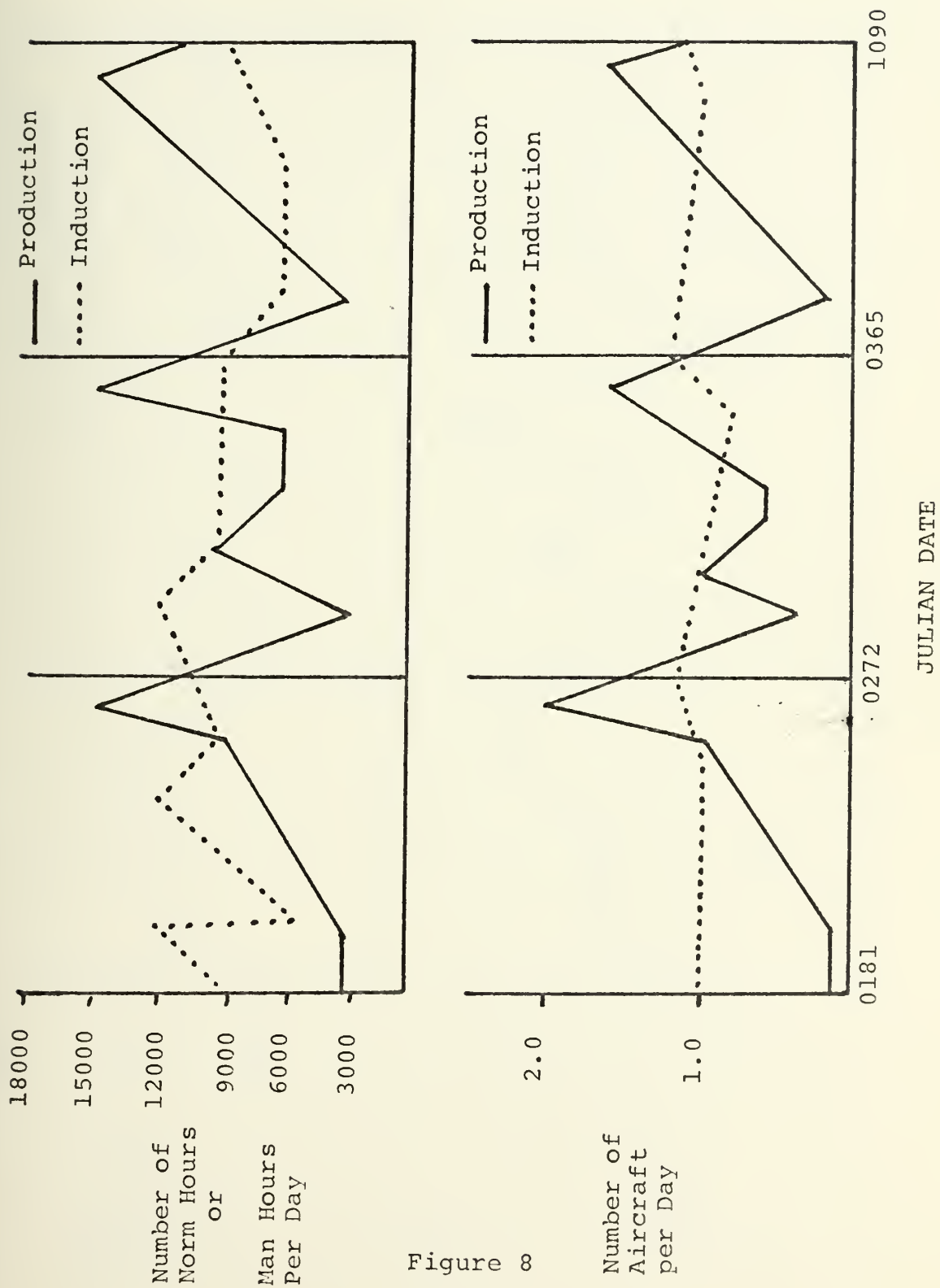


Figure 8





# ENGINE INDUCTION AND PRODUCTION RATES

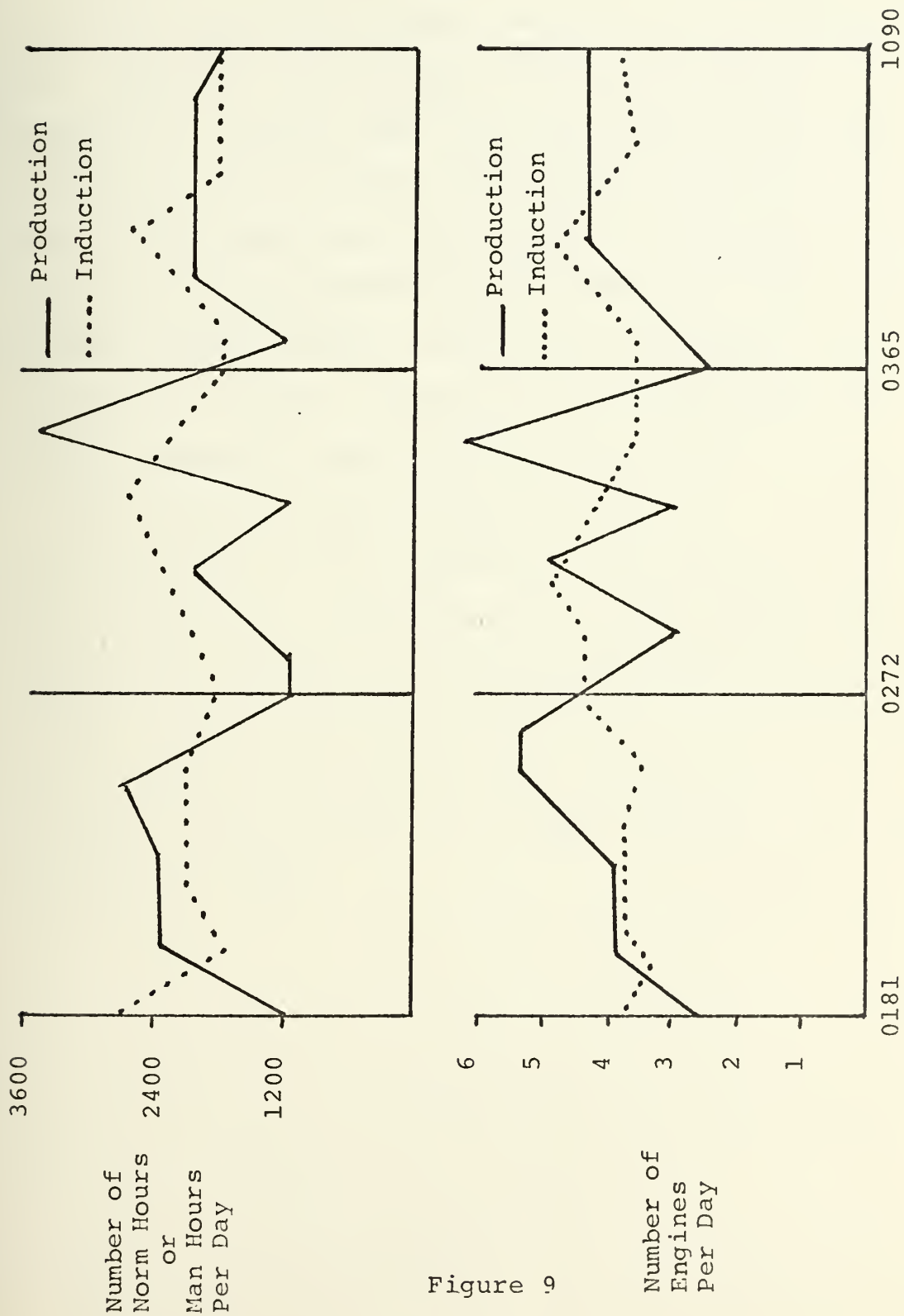


Figure 9



Number of jobs in shop versus time was plotted to confirm this relationship between induction rate and production rate. Number of jobs in shop should be increasing at the beginning of a quarter when production rate is less than induction rate and should be decreasing at the end of the quarter when production rate is greater than induction rate (Figure 10). This plot of jobs in shop versus time, with a maximum near mid-quarter and minimums just after the beginnings of a quarter, is consistent with the production rate and induction rate relationship described above.

The variable, number of jobs in shop ( $N$ ), was considered to be descriptive of the production and induction rates and, incidentally, also descriptive of shop congestion. By using number of jobs in shop as an independent variable in the Cobb-Douglas production function, it was possible at once to describe efficiencies due to specialization of personnel, inefficiencies due to shop congestion, and work flow through NARFNI.



NUMBER OF JOBS IN SHOP

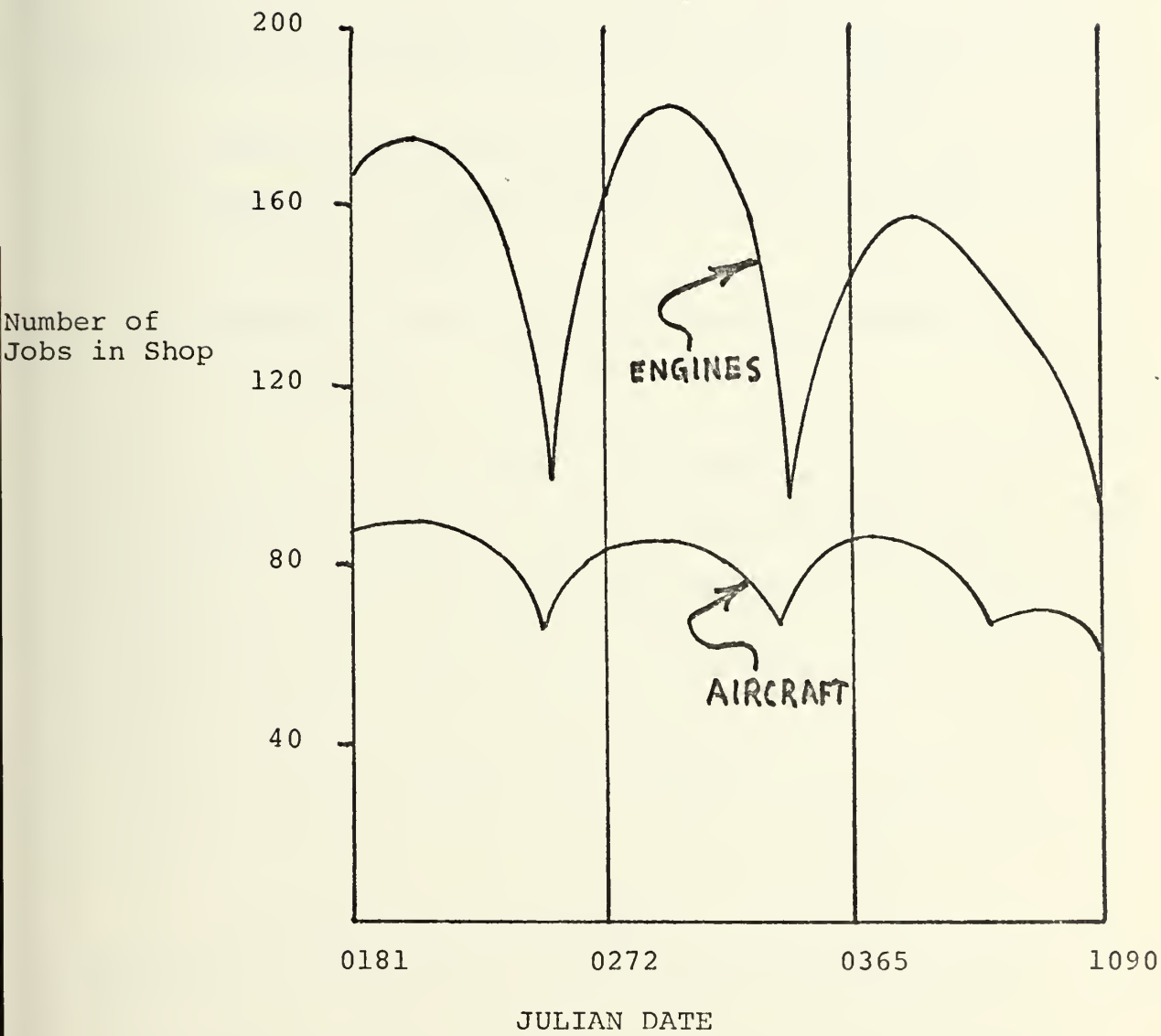


Figure 10



#### IV. THE ESTIMATION OF COBB-DOUGLAS PRODUCTION FUNCTIONS AND CONCLUSIONS

##### A. ESTIMATION OF PRODUCTION FUNCTIONS

Independent variables which were considered candidates for explaining aggregate hours of norm per day (APH) were

Aggregate prorated man hours expended per day (APL)

Aggregate prorated Direct Labor Dollars expended per day (APD)

Aggregate direct material dollars expended per day (APM)

However, early conversations with NARFNI personnel led to the inclusion of number of jobs in shop on each day (N) into the group of independent variables because it was believed to describe shop congestion and work flow. The Cobb-Douglas production function of this form is

$$APH = A (APL)^{\alpha} (APD)^{\beta} (APM)^{\gamma} (N)^{\delta} \quad (1)$$

A = a constant multiplier

Performing a logarithmic transformation on both sides:

$$\begin{aligned} \log APH = \log A + \alpha \log (APL) + \beta \log (APD) \\ + \gamma \log (APM) + \delta \log (N) \end{aligned} \quad (2)$$





The Biomedical Stepwise Regression Program, BMD02R, was used to perform the regression analysis. (Dixon, 1971) This program computes a series of multiple linear regressions in a stepwise manner. At each step one variable is added or removed from the regression equation. The variable added is the one which makes the greatest reduction in the error sum of squares between the data points and the regression plane. Equivalently it is the variable which has the highest partial correlation with the dependent variable partialled on the variables which have already been added; and it is the variable which, if added, would have the highest F value. A variable is removed when its F value becomes too low. The stepwise procedure is terminated when all variables are included in the regression equation, or the variables which are not included, would produce F values below a preselected limit. For this study, the F-level required for inclusion was 0.01, and the F-level for deletion was 0.0005.

#### 1. The Aircraft Rework Program

A regression was performed on the available aircraft production data, aggregated and combined as in equations (1) and (2) above. The regression results follow in Table I. Available data for the aircraft and engine programs, because of NARFNI recording procedures, did not include items already in shop at the beginning of the data period and items partially



TABLE I. STEPWISE REGRESSION OF AGGREGATED AIRCRAFT PRODUCTION DATA

STEP	1	2	3	4
Entered Previously Entered	N	APM N	APL N APM	APD N APM APL
R	0.9343	0.9502	0.9610	0.9921
Standard Error of Est.	0.0285	0.0250	0.0222	0.0101
COEFFICIENTS				
Constant	4.40108	5.57450	4.04980	2.96810
N	1.04987	1.20287	0.88208	0.02855
APM		-0.18588	-0.23714	-0.01813
APL			0.38104	2.96681
APD				-1.90684
Sum of Var. Coef.	1.04987	1.11699	1.02598	1.07039
STANDARD ERROR OF COEFFICIENTS				
N	.02399	0.02671	0.04410	0.03298
APM		0.02004	0.01879	0.01087
APL			0.04414	0.08182
APD				0.05849
F TO REMOVE AN INDEPENDENT VARIABLE				
N	1914.8035	2028.0881	399.9829	0.7490
APM		85.9980	159.3094	2.7803
APL			74.5148	1314.9087
APD				1062.8479
REGRESSION/RESIDUAL ANALYSIS OF VARIANCE				
F-ratio	1914.803	1292.076	1114.008	4304.031
df	1/279	2/278	3/277	4/276
F(tab., .05)	3.88	3.03	2.66	2.40



completed and still in shop at the end of the data period. Therefore, plots of norm, resources and jobs in shop against time in calendar days began at zero, rose to normal levels, began quarterly fluctuations and then decreased to zero at the end of the data period. The transits between zero and the normal levels were artificial since the entire in-process inventory at the beginning and end of the data period was not reflected in the data. If the entire in-process inventory had been included, the plots would have begun and ended at normal levels. To account for this anomaly, plots of the variables were studied. These plots indicated that there was an interval at the center of the data period, free from the tail effects at the ends, in which data would be accurate. This period was determined to be from day one hundred and forty to day four hundred and twenty. Therefore, the data base produced two hundred eighty-one daily aggregate data points. The daily aggregate data is presented in Appendix A.

The multiple correlation coefficient,  $R$ , was high at each step and the standard error of the estimated relationships were small and relatively constant from step to step. At the second step of the regression process, prorated material costs (APM) joined number of jobs in shop (N) as an independent variable in the regression equation. The coefficient associated with APM was negative in sign and remained negative at each step thereafter.



Through Step 3, the standard error of the estimate of each coefficient was small relative to the absolute value of the estimate of the coefficients themselves; in each case the standard error was at least one-fifth the estimated coefficient. Thus, from the assumption of normality of the deviations from the regression plane, it was possible to imply that about two-thirds of the data points lie less than one standard error from the regression plane. Since the standard error was small in all cases, the regression planes estimated in steps 1 through 3 were shown to fit the prorated data base well. In step 4, however, the standard error of the estimated coefficients for N and APM were almost as large as the coefficients themselves. This indicated possible collinearity between N, APM, and the new variable APD.

The F statistic computed for each separate statistic in each step of the regression tests the null hypothesis:

$$H_0 : \beta_i = 0$$

$\beta_i$  = the coefficient of the  $i^{\text{th}}$  independent variable;  $\beta_0$  is the constant term.

This hypothesis should be accepted when the tabulated value of F at confidence level  $1 - \alpha$  (Theil, pp. 720-723, 1971) is smaller than the F-statistic computed for each coefficient at a specific step in the regression (Theil, pp. 139-191, 1971).





For the daily aggregated data, the tabulated F statistics at the .95 confidence level were

F		
1,279		
F		
1,278		
	≈	3.88
F		
1,277		
F		
1,276		

For steps one through three, the computed F statistics for each estimated coefficient were very much greater than the tabulated value; none are less than 74.5; thus, the null hypothesis is rejected in each case. Therefore, at each step each coefficient included in the regression equation was found to be significantly different from zero at the .95 level; and therefore, the corresponding independent variable was assumed significant in explaining the variance of the dependent variable. At step four, however, with the inclusion of prorated labor costs as an independent variable, the computed F statistics for both N and APM were both less than the tabulated value at the .95 level. This led to the acceptance of the null hypothesis, indicating that, at step four, the once highly significant variables, N and APM, were no longer significant. These results seemed to indicate that the inclusion of labor costs created a collinear set of



independent variables. Therefore, the step four regression equation was assumed not to explain the structural relationship between available resources, work flow and shop congestion nearly as well as the regression in the previous steps.

The analysis of the variance (ANOVA) of the regression relationship and the variance of the residuals compare an F statistic computed for the overall regression relationship with a tabulated F statistic at level  $1 - \alpha$ .

$$F_{i,j}$$

i = number of degrees of freedom of the regression  
j = number of degrees of freedom of the residuals

This procedure tests the null hypothesis:

$$H_0 : \beta = 0$$

$$\beta = \begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \vdots \\ \beta_m \end{pmatrix}$$

$\beta_0$  = constant term  
 $m$  = number of independent variables in the regression equation



This procedure, like the F-test for a single coefficient, accepts the null hypotheses when  $F_{\text{computed}}$  is less than  $F_{\text{tabulated}}$ , and rejects the hypothesis when  $F_{\text{computed}}$  is larger.

At each step in the regression, the computed F-statistic is at least four hundred times greater than the tabulated F-statistic. Therefore, the overall regression relationship at each step was found to provide a good estimate of the dependent variable.

Recall the object of the study was to describe the structure of the production process so that trade-offs between resources could be examined; it was not to find the most accurate predictor of the dependent variable, hours of norm. As long as the dependent variable was estimated reasonably well, it was much more important to realize gains in efficiency in describing structure, rather than in describing the dependent variable better. Since it was shown by F-tests on the individual coefficients that the step four regression did not describe the structure of the production process well, and since the overall F-tests showed that the regressions at each step estimated the dependent variable very well, it was decided to choose the regression at step three as the superior regression, even though the overall F-test at step four indicated that the step four regression described the dependent variable best.



The fact that APM entered the regression with a negative sign and remained negative throughout the regression ran counter to any prior understanding of the production process. Two possible explanations were:

Collinearity appeared with APM at step two; or

Funds were allocated to the purchase of material past the point of positive returns.

If the latter were true, some of the funds expended on material should be redirected to labor. In an attempt to resolve this question, APD was dropped as a candidate for an explanatory variable, as it produced collinearity; and the daily aggregated data was further aggregated over a seven-day period. Thus the data set was reduced from two hundred eighty-one points to forty points.

$$A_d = \begin{cases} APH_d \\ \text{or} \\ APM_d \\ \text{or} \\ APL_d \end{cases}$$

$$W_t = \begin{cases} WPH_t \\ \text{or} \\ WPM_t \\ \text{or} \\ WPL_t \end{cases}$$

$$W_t = \sum_{i: 7(t-1)+1}^{7(t-1)+7} A_i \quad t = (1, \dots, 40) \quad (3)$$

$$W_n = N_{7t} \quad t = (1, \dots, 40)$$





A stepwise regression was performed as before, and the results follow in Table II. Weekly resource variables entering in an order different from the daily variables in the preceeding regression indicated a change in the incremental explanatory power of each variable as the data was aggregated from periods of one day to one week.

The multiple correlation coefficients were high at each step of the regression but generally not as high as those for the daily aggregated data. The standard errors of the estimated regression relationships were small and relatively constant. All standard errors of the individual coefficients were small, less than one-fourth of the respective estimates of the coefficients. The computed F-statistics were all approximately four times the computed F-statistic at the .95 level. This implied that the null hypothesis,  $\beta_i = 0$ , should be rejected; thus, all independent variables included at each step were significant in explaining the dependent variable. At each step, the computed F-statistics for the overall regression relationships were at least thirty times the tabulated F-statistic at the .95 level. This implied that the null hypothesis,  $\beta = 0$ , should be rejected; thus, at each step, the regression relationship described the dependent variable very well.

Weekly aggregate material costs entered the regression with negative sign just as they did for the daily aggregate data.



TABLE II. STEPWISE REGRESSION OF AGGREGATED AIRCRAFT PRODUCTION DATA  
SUMMED OVER SEVEN-DAY PERIODS (APD OMITTED)

STEP	1	2	3
Entered	N	WPL N	WPM N
Previously Entered			WPL
R	0.8804	0.9178	0.9451
Standard Error of Est.	0.0371	0.0315	0.0262
COEFFICIENTS			
Constant	6.81766	3.25141	4.02900
N	0.94447	0.54866	0.57613
WPL		0.48279	0.66744
WPM			-0.24639
Sum of Var. Coef.	0.94447	1.03145	0.99718
STANDARD ERROR OF COEFFICIENTS			
N	0.08252	0.12180	0.10184
WPL		0.12156	0.11080
WPM			0.05951
F TO REMOVE AN INDEPENDENT VARIABLE			
N	131.0046		32.0017
WPL		20.2927	36.2873
WPM		15.7742	17.1440
REGRESSION/RESIDUAL ANALYSIS OF VARIANCE			
F-ratio	131.004	98.956	100.374
df	1/38	2/37	3/36
F(tab., .05)	4.10	3.26	2.86



Also, in the case of weekly data, the F-statistic for both the individual coefficients and the overall regression relationships were generally smaller than those for the daily data. It seemed therefore that nothing was gained by aggregating the daily data over seven-day periods; the negative sign associated with material cost was not removed, and loss of power of estimation was indicated.

As a further check, however, the weekly data was aggregated over two-week periods. The variables WPH, WPM and WPL, were transformed into bi-weekly variables, BPH, BPM and BPL respectively; and the data set was reduced from forty to twenty observations. Again a stepwise regression was performed, and the results follow in Table III. The results were almost exactly analgous to the cases for daily and weekly data. There was some indication of collinearity when N entered the regression at step two with an F-statistic implying acceptance at the .95 level of the null hypothesis,  $\beta_N = 0$ ; N then, however, became slightly significant again in step three. In general, the power of estimation of the regression on bi-weekly data was distinctly less than those of the regressions on daily and weekly data. Again material cost entered the regression with a negative sign.

In summary, aggregation of daily data did not increase estimative power of the regression, nor did the problem of possible collinearity become resolved. Therefore,



TABLE III. STEPWISE REGRESSION OF AGGREGATED AIRCRAFT PRODUCTION DATA  
SUMMED OVER FOURTEEN-DAY PERIODS (APD OMITTED)

STEP	1	2	3
Entered Previously Entered	BPL	N BPL	BPM BPL N
R	0.8522	0.8817	0.9155
Standard Error of Est.	0.0395	0.0367	0.0322
COEFFICIENTS			
Constant	0.09737	2.20651	3.07491
BPL	0.91695	0.70340	0.89335
N		0.28122	0.31379
BPM			-0.25692
Sum of Var. Coef.	0.91698	0.98462	0.95022
STANDARD ERROR OF COEFFICIENTS			
BPL	0.13272	0.16375	0.16348
N		0.14125	0.12565
BPM			0.10486
F TO REMOVE AN INDEPENDENT VARIABLE			
BPL	47.7372	18.4520	29.8601
N		3.9140	6.2366
BPM			6.0033
REGRESSION/RESIDUAL ANALYSIS OF VARIANCE			
F-ratio	47.737	29.690	27.619
df	1/18	2/17	3/16
F(tab., .05)	4.41	3.59	3.24





TABLE III. STEPWISE REGRESSION OF AGGREGATED AIRCRAFT PRODUCTION DATA  
SUMMED OVER FOURTEEN-DAY PERIODS (APD OMITTED)

STEP	1	2	3
Entered	BPL	N	BPM
Previously Entered		BPL	BPL
			N
R	0.8522	0.8817	0.9155
Standard Error of Est.	0.0395	0.0367	0.0322
COEFFICIENTS			
Constant	0.09737	2.20651	3.07491
BPL	0.91695	0.70340	0.89335
N		0.28122	0.31379
BPM			-0.25692
Sum of Var. Coef.	0.91698	0.98462	0.95022
STANDARD ERROR OF COEFFICIENTS			
BPL	0.13272	0.16375	0.16348
N		0.14125	0.12565
BPM			0.10486
F TO REMOVE AN INDEPENDENT VARIABLE			
BPL	47.7372	18.4520	29.8601
N		3.9140	6.2366
BPM			6.0033
REGRESSION/RESIDUAL ANALYSIS OF VARIANCE			
F-ratio	47.737	29.690	27.619
df	1/18	2/17	3/16
F(tab., .05)	4.41	3.59	3.24



TABLE IV. STEPWISE REGRESSION OF AGGREGATED ENGINE PRODUCTION DATA

Entered Previously Entered	STEP			
	1	2	3	4
	APL	APM APL	N APL APM	APD APL APM N
R	0.9813	0.9933	0.9938	0.9941
Standard Error of Est.	0.0290	0.0174	0.0168	0.0164
COEFFICIENTS				
Constant	-0.43166	-1.89837	-2.06516	-1.72979
APL	1.05883	0.92210	1.06699	1.31147
APM		0.24135	0.19621	0.19092
N			-0.08021	-0.10566
APD				-0.21453
Sum of Var. Coef.	1.05883	1.17345	1.18199	1.18220
STANDARD ERROR OF COEFFICIENTS				
APL	0.01243	0.00965	0.03303	0.06985
APM		0.01127	0.01625	0.01589
N			0.01754	0.01827
APD				0.05439
F TO REMOVE AN INDEPENDENT VARIABLE				
APL	7259.0938	9124.1641	1043.5493	352.5334
APM		497.1812	145.8253	144.3027
N			20.9104	33.4314
APD				15.5556
REGRESSION/RESIDUAL ANALYSIS OF VARIANCE				
F-ratio	7259.102	10333.031	7389.027	5836.867
df	1/279	2/278	3/277	4/276
F(tab., .05)	3.88	3.03	2.66	2.40



compared with a tabulated value of 3.38 at the .95 level. Therefore, each explanatory variable remains significant at each step in the regression. There was some suggestion of collinearity, however, when N entered the regression in step three. The F-statistic for APL was reduced by a factor of nine and the F-statistic for APM reduced by a factor of three. There was further indication of collinearity at step four when APD entered and the F-statistic for APL was reduced again by a factor of three. However, the computed F-statistics for the overall regression relationships were never less than two hundred forty times larger than the tabulated F-statistics at the .95 level. Therefore, the dependent variable was considered to be described extremely well at each step.

As mentioned above, there was a suggestion of possible collinearity at the third step of the regression when N entered the regression, negative in sign. There was no prior understanding to apply to interpret this negative sign. It could have been because of a near collinear relationship; or it could have been the result of shop congestion. Shop congestion describes a situation in which there are simply too many engine jobs in shop relative to the size of the work force. Work then could progress on several jobs at one time when it might have been more efficient to concentrate the same work force on fewer jobs, finishing them sooner. To investigate the question, the daily data was aggregated into weekly



data as in equations (3) and (4). APD was omitted, as it was assumed to create collinearity.

Again, a stepwise regression was performed, regressing WPM, WPL and WN on WPH. The results follow in Table 5. Again, the correlation coefficients were high and the standard errors low and relatively constant at each step in the regression. Also N entered again at step three with negative sign; but this time with the standard error of its associated coefficient (.042) greater than the estimated coefficient (.030) and the computed F-statistic (0.51) less than the tabulated F-statistic (4.11) at the .95 level. This indicated that N described the weekly data poorly and was not significant in the regression when it entered. The F-statistics of the overall regression relationships at each step remained high, at least four hundred fifty times the tabulated F-statistic at the .95 level. The regression on the bi-weekly data tended to confirm the hypothesis that N introduces collinearity to the set of explanatory variables.

As a further test, the weekly data was further aggregated over two-week periods, reducing the data set to twenty points. A stepwise regression was again performed and the results follow in Table VI.

The multiple correlation coefficients were again high and the standard errors of the estimated relationships low and relatively constant. All standard errors of the





TABLE V. STEPWISE REGRESSION OF AGGREGATED ENGINE PRODUCTION DATA  
SUMMED OVER SEVEN-DAY PERIODS (APD OMITTED)

STEP	1	2	3
Entered			
Previously Entered	WPL	WPM WPL	N WPL WPM
R	0.9803	0.9938	0.9939
Standard Error of Est.	0.0287	0.0164	0.0165
COEFFICIENTS			
Constant	-0.55639	-2.39041	-2.51348
WPL	1.05990	0.92463	0.98232
WPM		0.26229	0.23858
N			-0.03035
Sum of Var. Coef.	1.05990	1.18692	1.19055
STANDARD ERROR OF COEFFICIENTS			
WPL	0.03462	0.02483	0.08419
WPM		0.02928	0.04428
N			0.04230
F TO REMOVE AN INDEPENDENT VARIABLE			
WPL	937.4575	1387.1077	136.1376
WPM		80.2544	29.0358
N			0.5148
REGRESSION/RESIDUAL ANALYSIS OF VARIANCE			
F-ratio	937.449	1486.443	978.139
df	1/38	2/37	3/36
F(tab., .05)	4.10	3.26	2.86



TABLE VI. STEPWISE REGRESSION OF AGGREGATED ENGINE PRODUCTION DATA  
SUMMED OVER FOURTEEN-DAY PERIODS (APD OMITTED)

STEP	1	2	3
Entered	BPL	BPM	N
Previously Entered		BPL	BPL BPM
R	0.9802	0.9945	0.9946
Standard Error of Est.	0.0278	0.0152	0.0154
COEFFICIENTS			
Constant	-0.48957	-2.51588	-2.25042
BPL	1.04933	0.92428	0.82224
BPM		0.26226	0.30629
N			0.04877
Sum of Var. Coef.	1.04933	1.18654	1.17730
STANDARD ERROR OF COEFFICIENTS			
BPL	0.04999	0.03327	0.14386
BPM		0.03985	0.07262
N			0.06683
F TO REMOVE AN INDEPENDENT VARIABLE			
BPL	440.5701	771.8689	32.6671
BPM		43.3171	17.7883
N			0.5324
REGRESSION/RESIDUAL ANALYSIS OF VARIANCE			
F-ratio	440.571	759.823	492.794
df	1/18	2/17	3/16
F(tab., .05)	4.41	3.59	3.24



separate regression coefficients were small relative to the coefficient estimates themselves except for the entry of N into the regression at the third step. Also at this step, the computed F-statistic of N (0.53) was less than the tabulated F-statistic (4.49) at the .95 level. This again indicates the independent variable explains the dependent variably poorly and that it is insignificant when it enters the regression. However, when it does enter at the third step, it enters with positive sign. This change in sign tends to confirm a high variance of the estimate of the coefficient of N at the third step, suggesting collinearity.

The recurring instances of N having large standard error and being insignificant in the regression indicated that it should be omitted as an independent variable from the regression relationship. Since the daily aggregated data produced generally higher significance levels in the estimates, the regressions on the daily aggregated data seemed superior to the regressions on weekly and bi-weekly data. Specifically, the second step in the daily aggregated data seemed to best estimate the dependent variable and describe the structure of the production process.

### 3. The Component Repair Program

A stepwise regression was performed on the available production data for the component repair program. The only available data gave average norm per unit per month, and average man hours and material dollars expended per unit per



month, and number of units completed each month. In particular, no information was available indicating the kinds of components repaired and the degree of variance which may have existed in the resource requirements between types of components repaired. It was desired to massage the component data to be able to perform a regression somewhat analogous to those for the aircraft and engine programs. To get monthly product and resource usage data, the average unit product per month and average unit resource usage per month were multiplied by the number of units completed for a given month. This provided data on average monthly norm produced (MPH), average man hours expended per month (MPL), and average material dollars expended per month (MPM). Data was available for fifteen monthly observations. The monthly aggregate data is given in Appendix C. The Cobb-Douglas Production Function to be estimated was:

$$\text{MPH} = (A) (\text{MPL})^{\alpha} (\text{MPM})^{\beta} \quad (5)$$

taking logarithms of both sides:

$$\log \text{MPH} = \log A + \alpha \log (\text{MPL}) + \beta \log (\text{MPM}) \quad (6)$$

The stepwise regression results follow in Table VII.

The multiple correlation coefficients were fairly low, none greater than .60, which seemed to indicate poor performance of the regression estimates. The standard errors of the estimate were low and relatively constant, however.





TABLE VII. STEPWISE REGRESSION OF AGGREGATED COMPONENT PRODUCTION DATA

STEP	1	2
Entered	MPL	MPM
Previously Entered		MPL
R	0.5582	0.5988
Standard Error of Estimate	0.1505	0.1512
COEFFICIENTS		
Constant	1.38715	0.95767
MPL	0.88306	0.69785
MPM		0.18871
Sum of Variable Coefficients	0.88306	0.8556
STANDARD ERROR OF COEFFICIENTS		
MPL	0.36408	0.41562
MPM		0.20110
F TO REMOVE AN INDEPENDENT VARIABLE		
MPL	5.8829	2.8192
MPM		0.8806
REGRESSION/RESIDUAL ANALYSIS OF VARIANCE		
F-ratio	5.883	3.355
df	1/13	2/12
F(Tab., .05)	4.67	3.88



The standard errors of the coefficients of MPL in steps one and two were moderately small relative to the estimates of the coefficients themselves, but at step two, MPM entered with standard error (0.20) greater than its estimate (0.19). Also, MPL was slightly significant at step one, as its computed F-statistic (5.88) was greater than the tabulated F-statistic (4.67) at the .95 level; but when MPM entered at step two, the computed F-statistics for both MPL and MPM were less than the tabulated F-statistic at the .95 level, indicating both variables became insignificant at step two. Also, the computed F-statistic for the overall regression relationship was greater than the tabulated F-statistic at the .95 level at step one (5.88 greater than 4.67); but the computed value was less than the tabulated value at step two (3.35 less than 3.88), indicating the entire regression relationship was insignificant at step two.

Because of the unsatisfactory performance of the regression relationship when MPM entered the regression at step two, the regression relationship at step one was chosen as the only possible regression relationship more explicit than a single constant term. It must be noted, however, that this regression on component production data was not nearly as powerful as the previous regressions on aircraft and engine production data.



## B. CONCLUSIONS

### 1. The Aircraft Rework Program

Number of jobs in shop (N), daily aggregate prorater material costs (APM), and daily aggregate prorated man hours expended (APL), were found to be significant independent variables which explained the dependent variable aggregate prorated hours of norm produced (APH). The Cobb-Douglas Production Function estimated by the regression analysis was found to be:

$$\text{APH} = (57.36) (\text{APL})^{0.38104} (\text{APM})^{-0.23714} (\text{N})^{0.88208}$$

The negative sign on the exponent for APM indicated that there may be some misallocation of funds to purchase material which is not used expeditiously. The relationship suggested that an increase in labor's share of the available funds may increase daily production. The sum of the resource factor exponents in the production function being greater than one, indicated that NARFNI was operating its aircraft rework program at increasing returns to scale.

### 2. The Engine Repair Program

Daily aggregate prorated man hours expended (APL), and daily aggregate prorated material costs expended (APM) were found to be significant independent variables which explained the dependent variable, daily aggregate prorated



hours of norm produced (APH). The Cobb-Douglas Production Function estimated by regression analysis was found to be:

$$APH = (0.15)^{0.92210} (APL)^{0.25135} (APM)$$

The sum of the resource factor exponents in the production function being greater than one, indicated that NARFNI was operating its engine repair programs at increasing returns to scale.

### 3. The Component Repair Program

Monthly aggregate man hours expended (MPL) was found to be the only significant independent variable which was useful in explaining the dependent variable, monthly aggregate hours of norm produced (MPH). The Cobb-Douglas Production Function estimated by regression analysis was found to be:

$$MPH = (4.00)^{0.88306} (MPL)$$

The exponent of the only significant factor of production being less than one, indicated that NARFNI was operating its component repair program at decreasing returns to scale.





## V. AREAS FOR FURTHER STUDY

### A. FURTHER ANALYSIS OF PRODUCTION AND INDUCTION RATES

When induction rate was plotted against calendar days, induction rates for both aircraft and engines were seen to be quite constant over time. Production rate plotted similarly indicated that production rate was low just after the beginning of a calendar quarter and increased to its highest point at the end of a quarter. It was shown that this implied a shop load curve with a maximum near mid-quarter and minimums near the beginning and ending of a quarter.

As a further check, number of days in shop was plotted against production date for the aircraft program. It was anticipated that for each type of aircraft, the aircraft produced nearer the end of the quarter would have spent the shortest time at NARFNI. Instead, it was found that the length of time spent at NARFNI for an aircraft was independent of the position of its production date in the quarter. Two possible explanations for this are suggested. First, the increasing production at the end of the quarter may be a function of the scheduling of different types of aircraft into NARFNI for rework. Since different types of aircraft vary greatly in degree of complexity, the average time required for rework for the different types also varies greatly. The various types of aircraft could be arriving at NARFNI in



combinations such that the inductions are distributed uniformly, but in such a way that aircraft tend to arrive one average repair time before the end of a quarter. Second, NARFNI may tend to use the period immediately after the beginning of a quarter to work on extremely large jobs, possibly those routinely stretching into the next quarter or over several quarters. Then, as time till the end of the quarter grows shorter, effort is shifted to those aircraft which can be completed that quarter.

It is not clear if either of these possibilities is a correct explanation for the increasing production rate at the end of a quarter, but the question deserves investigation.

#### B. DISTRIBUTED LAG MODELS

Because production rate increases at the end of a quarter, the assumption that labor and material are applied uniformly to a job may not adequately describe the actual situation. If the analysis of increasing production rate over a quarter indicates that actual production effort varies over the quarter, some distributed lag model for the aggregation of products and resources would seem appropriate. (Theil, pp. 258-259, 1971).

#### C. MULTIPLE EQUATION MODELS

It is likely that further investigation will show that production rate is a function of shop load. If there are too



few jobs in shop, the labor force may not be used to its full extent; if there are too many jobs in shop, effective allocation of labor and material may become difficult.

Then perhaps

$$p = f(n, t)$$

$p$  = production rate  
 $n$  = number of jobs in shop  
 $t$  = time till end of quarter

But

$$n = g(p_1, i)$$

$i$  = exogenous induction rate  
 $p_1$  = lagged production rate

Therefore

$$p = f[g(p_1, i), t]$$

And the two equations would be

$$p = f[g(p_1, i), t]$$

and

$$APH = k(APL, APM, p)$$

Multiple equation regression techniques could then be used. (Theil, Chapter 9, 1971).

#### D. ANALYSIS OF TECHNOLOGICAL CHANGE (Solow, 1968)

Let

$Q$  = output or product

$L$  = labor resource input

$K$  = capital resource input

$t$  = continuous time variable



Define "technicological change" to indicate any kind of shift in the production function.

Assume that resource factors are paid their marginal product.

The production function can be given by

$$Q = F(K, L, t) \quad (1)$$

where  $t$  accounts for the cumulated effect of shifts in production over time.

Define

$$q = Q/L$$

$$k = K/L$$

$$W_L = (\partial Q / \partial L) (L/Q)$$

$$W_K = (\partial Q / \partial K) (K/Q)$$

Solow shows that with the above definitions and assumptions, equation (1) leads to

$$\frac{\dot{q}}{q} = \frac{1}{F} \frac{\partial F}{\partial t} + W_K \frac{\dot{k}}{k} \quad (2)$$

Where the dots indicate time derivatives

The production function is completely defined for any fixed  $t$  by a plot of  $q$  against  $k$ , but the plot is shifting in time. Therefore if we observe production at two points in time,  $t_1$  and  $t_2$ , the movement in the  $q, k$  plane, from  $P_1$  to  $P_2$  is compounded of

a movement from one curve to another with resources held constant at the level at  $t = t_1$





and

a movement along the new curve with resources changing to the level at  $t = t_2$

In Figure 11, for every value of  $k$ , the  $q$  value on the curve for  $t = t_2$  is greater than the corresponding point for  $t = t_1$ . There has been a technological increase in production over time. The problem is to estimate how much of the total change in production, over the time interval  $t_1$  and  $t_2$  is due to technological change and how much due to the change in the level of resources.

For small changes in  $k$ , the production curve at any time and any value of  $k$ , can be approximated by its tangent. In Figure 11, the line tangent to the production curve for  $t = t_2$  at  $P_2$  produces an approximate point  $P^*$  for the  $k_1$  level at  $t = t_1$ .

Thus

$$\frac{P^* - P_1}{q_1}$$

estimates the change in technological change at  $q_1$  and  $k_1$ . Solow shows that

$$\frac{P^* - P_1}{q_1} = \frac{\Delta q}{q} - W_K \frac{\Delta k}{k} \quad (3)$$

Emperical production data would be available from two time periods, one before WIPICS installation and the other



# TECHNOLOGICAL CHANGE OVER TIME

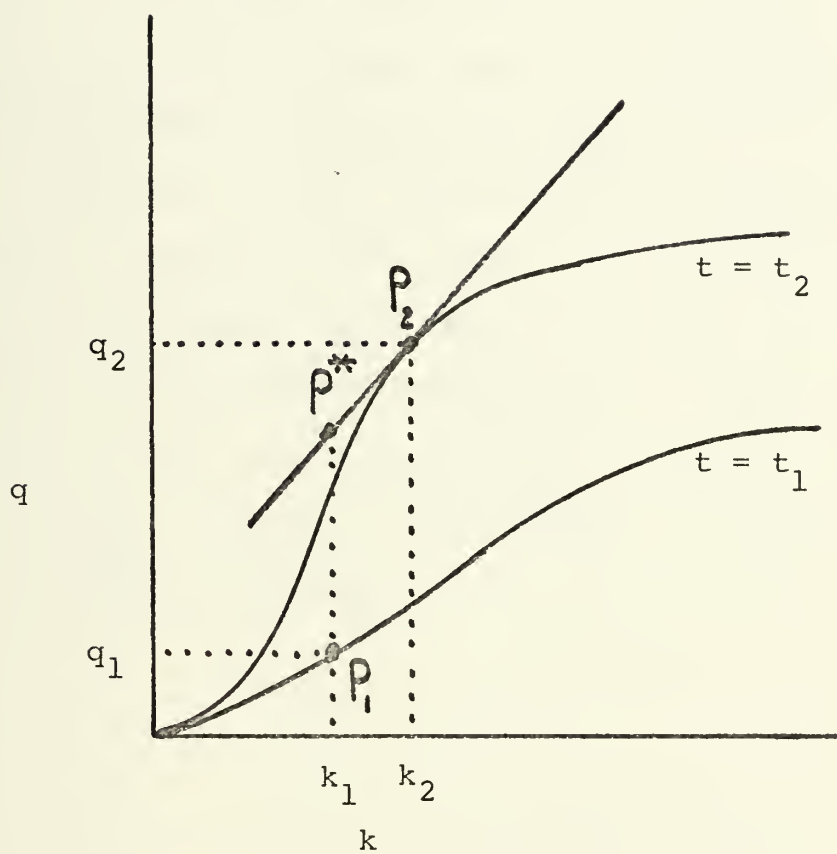


Figure 11



after installation. Average product and resource usage could be computed. With  $k$  defined as  $M/L$ , where  $M$  is the material resource, the differences indicated in equation (3) could be taken and two points on the  $k, q$  plane found. The two points thus obtained could then be compared by means of the above analysis and the degree of technological change during WIPICS installation estimated.

It is important to note that the above procedure identifies all change in production, not just change in production attributable to WIPICS installation, but if plant conditions such as size and composition of work force, overtime policy or shop load remains fairly constant, the above procedure should provide a rough estimate of the impact of WIPICS on NARFNI production.



DAILY AGGREGATED AIRCRAFT  
PRODUCTION DATA  
(JULIAN DATE 0069 TO 1235)

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1483.788	1518.984	8999.180	3079.188	15
1483.788	1518.934	8999.180	3079.188	15
1582.049	1612.836	9555.762	3311.918	16
1582.049	1612.836	9555.762	3311.918	16
1582.049	1612.836	9555.762	3311.918	16
1651.063	1680.378	9965.730	3461.024	17
1651.063	1680.378	9965.730	3461.024	17
1651.063	1680.378	9965.730	3461.024	17
1651.063	1680.378	9965.730	3461.024	17
1651.063	1680.378	9965.730	3461.024	17
1651.063	1680.378	9965.730	3461.024	17
1651.063	1680.378	9965.730	3461.024	17
1651.063	1680.378	9965.730	3461.024	17
1651.063	1680.378	9965.730	3461.024	17
1842.822	1867.384	11075.609	3906.268	19
1932.566	1976.007	11713.566	4192.137	20
1932.566	1976.007	11713.566	4192.137	20
1932.566	1976.007	11713.566	4192.137	20
1932.566	1976.007	11713.566	4192.137	20
1932.566	1976.007	11713.566	4192.137	20
2025.589	2062.914	12228.609	4381.152	21
2025.589	2062.914	12228.609	4381.152	21
2128.220	2182.743	12914.508	5203.488	22
2315.495	2371.492	14034.941	5563.305	24
2315.495	2371.492	14034.941	5563.305	24
2315.495	2371.492	14034.941	5563.305	24
2315.495	2371.492	14034.941	5563.305	24
2388.538	2440.719	14456.488	5744.660	25
2456.720	2512.521	14874.773	5873.254	26
2456.720	2512.521	14874.773	5873.254	26
2525.572	2575.948	15242.738	5992.383	27
2525.572	2575.948	15242.738	5992.383	27
2525.572	2575.948	15242.738	5992.383	27
2525.572	2575.948	15242.738	5992.383	27
2525.572	2575.948	15242.738	5992.383	27
2525.572	2575.948	15242.738	5992.383	27
2612.725	2680.453	15875.176	6334.922	28
2612.725	2680.453	15875.176	6334.922	28
2790.035	2881.923	17107.883	6881.094	30
2961.252	3035.070	18015.773	7171.316	32
2961.252	3035.070	18015.773	7171.316	32
2961.252	3035.070	18015.773	7171.316	32
2961.252	3035.070	18015.773	7171.316	32
3153.699	3257.427	19323.617	8265.570	34
3365.238	3462.305	20531.270	8600.781	36
3445.116	3556.635	21112.250	8901.926	37
3534.478	3637.677	21577.930	9082.563	38
3640.217	3761.445	22327.113	9497.324	39
3640.217	3761.445	22327.113	9497.324	39
3640.217	3761.445	22327.113	9497.324	39
3756.739	3892.697	23106.262	9861.344	40
3956.740	4100.816	24338.426	10467.125	42
3956.740	4100.816	24338.426	10467.125	42
4096.852	4232.047	25119.770	10750.586	44
4196.910	4346.094	25819.883	11056.293	45
4196.910	4346.094	25819.883	11056.293	45
4196.910	4346.094	25819.883	11056.293	45
4276.152	4429.543	26301.430	11265.141	46
4276.152	4429.543	26301.430	11265.141	46
4276.152	4429.543	26301.430	11265.141	46
4351.516	4499.180	26706.859	11404.234	47
4460.180	4623.855	27455.359	11789.629	48
4460.180	4623.855	27455.359	11789.629	48
4460.180	4623.855	27455.359	11789.629	48
4681.395	5033.723	29951.910	12999.145	53
4993.270	5153.070	30686.105	13281.781	55
5374.750	5522.109	32895.715	14513.531	58
5490.023	5634.953	33594.090	14870.406	59
5490.023	5634.953	33594.090	14870.406	59
5490.023	5634.953	33594.090	14870.406	59
5490.023	5634.953	33594.090	14870.406	59
5601.137	5715.012	34053.809	15146.738	60
5735.004	5844.313	34847.629	15458.398	61
5915.184	6012.316	35861.113	16003.340	63
6024.391	6125.828	36550.590	16272.898	64
6094.563	6183.130	36882.500	16475.059	65
6094.563	6183.130	36882.500	16475.059	65
6094.563	6183.130	36882.500	16475.059	65
6094.563	6183.130	36882.500	16475.059	65



6213.133	6304.102	37634.324	16731.867	66
6369.953	6416.863	38319.313	17386.520	67
6574.512	6627.959	39612.547	17887.297	69
6643.477	6682.459	39938.609	18117.945	70
6769.227	6808.570	40723.172	18389.277	71
6769.227	6808.570	40723.172	18389.277	71
6769.227	6808.570	40723.172	18389.277	71
6902.336	6888.977	41261.578	18701.234	72
7020.902	7001.809	41961.660	18976.074	73
7079.727	7048.938	42250.840	19089.574	74
7200.016	7171.340	43014.355	19347.750	75
7149.566	7073.809	42469.852	19691.938	75
7149.566	7073.809	42469.852	19691.938	75
7149.566	7073.809	42469.852	19691.938	75
7239.742	7161.762	43006.512	19945.895	76
7297.449	7212.699	43373.684	20146.188	76
7355.117	7284.527	43838.543	20263.023	76
7610.645	7519.102	45273.895	20801.684	78
7568.270	7540.158	45320.254	20723.059	78
7568.270	7540.158	45320.254	20723.059	78
7568.270	7540.158	45320.254	20723.059	78
7775.852	7716.352	46405.879	21161.809	80
7775.852	7716.352	46405.879	21161.809	80
7967.742	7806.145	47014.922	20892.512	81
8083.895	7894.070	47587.004	21162.078	82
8232.520	8055.828	48445.691	21421.242	83
8232.520	8055.828	48445.691	21421.242	83
8232.520	8055.828	48445.691	21421.242	83
8559.051	8336.512	50142.730	21916.145	86
8381.922	8182.256	49246.855	21472.383	84
8505.477	8307.930	50013.910	21721.742	85
8621.516	8432.277	50797.520	21896.660	86
8260.527	8055.172	48608.125	20652.844	82
8260.527	8055.172	48608.125	20652.844	82
8260.527	8055.172	48608.125	20652.844	82
8373.570	8148.270	49199.270	20910.043	83
8603.742	8362.063	50526.516	21384.180	85
8609.102	8338.816	50393.977	21451.895	85
8636.203	8407.231	50813.004	21239.492	86
8637.930	8409.578	50838.293	21306.672	86
8637.930	8409.578	50838.293	21306.672	86
8637.930	8409.578	50838.293	21306.672	86
8877.926	8634.305	52213.129	21718.336	88
8950.680	8717.977	52743.754	21587.836	89
8872.719	8641.773	52308.477	21123.090	88
8915.480	8690.141	52624.117	21185.895	88
8849.984	8626.719	52247.363	21033.188	87
8849.984	8626.719	52247.363	21033.188	87
8849.984	8626.719	52247.363	21033.188	87
8972.152	8613.102	52271.516	20963.145	87
9074.980	8720.348	52912.891	21275.102	88
9028.191	8735.715	53037.168	21178.109	88
9289.977	8989.008	54583.023	21652.570	90
9289.977	8989.008	54583.023	21652.570	90
9219.801	8931.656	54251.113	21450.418	89
9119.734	8817.605	53551.004	21144.703	88
8683.719	8366.645	50810.930	20039.770	84
8922.813	8568.658	52012.531	20658.820	86
8838.301	8430.199	51171.941	20250.879	85
8988.301	8569.418	52009.137	20466.859	86
9162.898	8758.959	53167.191	20796.953	88
8910.477	8513.020	51683.879	19878.930	85
8910.477	8513.020	51683.879	19878.930	85
9059.762	8661.641	52603.688	20206.777	86
9238.848	8838.734	53668.113	20825.820	88
9044.066	8637.441	52435.184	20215.813	86
9053.363	8649.777	52532.898	19862.938	86
9167.797	8767.730	53273.402	20038.852	87
9167.797	8767.730	53273.402	20038.852	87
9167.797	8767.730	53273.402	20038.852	87
8630.863	8279.504	50271.137	18907.875	82
8882.586	8573.859	52060.035	19480.191	84









8548.063	8808.074	54474.195	20028.090	84
8548.063	8808.074	54474.195	20028.090	84
8548.063	8808.074	54474.195	20028.090	84
8660.563	8910.297	55103.637	20877.309	85
8580.230	8812.512	54526.453	20713.324	84
8672.754	8959.934	55507.426	20590.309	85
8526.184	8808.938	54573.688	19813.422	83
8612.555	8916.918	55210.281	20111.793	84
8566.016	8877.199	55044.109	20157.801	84
8511.195	8820.953	54696.379	20032.141	83
8511.195	8820.953	54696.379	20032.141	83
8817.711	9133.148	56682.520	20634.762	86
8617.320	8955.139	55604.316	19996.313	84
8463.766	8827.840	54860.160	19858.434	83
8363.227	8733.238	54348.309	19517.367	82
8234.066	8617.129	53639.926	19184.160	81
7766.703	8159.988	50833.293	18086.363	77
7766.703	8159.988	50833.293	18086.363	77
7733.668	8130.324	50749.395	18075.723	76
7810.098	8200.855	51210.426	18644.590	77
7613.500	8000.398	49984.031	18276.793	75
7613.500	8000.398	49984.031	18276.793	75
7613.500	8000.398	49984.031	18276.793	75
7429.555	7834.945	48940.563	17804.922	73
7519.383	7913.063	49431.234	18036.578	74
7388.949	7785.016	48631.641	17837.406	73
7544.215	7956.555	49749.141	18160.059	75
6772.113	7113.695	44661.637	16467.852	69
6772.113	7113.695	44661.637	16467.852	69
6772.113	7113.695	44661.637	16467.852	69
6772.113	7113.695	44661.637	16467.852	69
7048.438	7413.473	46557.695	17359.570	72
7161.508	7540.793	47364.102	17611.410	73
7243.172	7622.105	47874.551	17867.563	74
7438.965	7870.594	49475.586	18378.148	76
7406.438	7848.438	49355.453	18270.633	76
7406.438	7848.438	49355.453	18270.633	76
7406.438	7848.438	49355.453	18270.633	76
7406.438	7848.438	49355.453	18270.633	76
7649.543	8097.172	50954.266	19421.484	78
7649.543	8097.172	50954.266	19421.484	78
7826.148	8278.758	52103.715	19944.473	80
7926.148	8397.160	52879.266	20215.953	81
7898.340	8368.992	52709.270	20246.281	81
7898.340	8368.992	52709.270	20246.281	81
7898.340	8368.992	52709.270	20246.281	81
7932.313	8430.715	53122.816	20584.238	82
7922.320	8422.383	53110.563	20707.258	82
7868.238	8383.934	52900.945	20735.574	82
7976.813	8500.676	53664.730	21087.859	83
8176.438	8705.406	54997.328	21647.980	85
8176.438	8705.406	54997.328	21647.980	85
8176.438	8705.406	54997.328	21647.980	85
8349.254	8898.676	56252.879	22063.266	86
8455.086	9023.938	56972.352	22605.688	87
8583.652	9167.750	57919.980	22938.922	88
8716.422	9293.219	58752.266	23125.293	89
8733.172	9287.309	58785.898	23201.668	89
8538.520	9069.605	57476.176	22611.418	86
8446.301	8974.648	56895.590	22420.191	85
8576.863	9104.703	57748.836	22873.738	86
8514.352	9040.945	57358.461	22574.852	85
8585.461	9107.277	57780.438	22894.828	86
8783.340	9336.254	59245.590	23484.680	88
8847.340	9400.012	59658.547	23761.492	89
8847.340	9400.012	59658.547	23761.492	89
8847.340	9400.012	59658.547	23761.492	89
8706.895	9263.418	58800.762	22889.387	87
8481.938	9043.699	57455.117	21692.348	85
8481.938	9043.699	57455.117	21692.348	85
8795.820	9374.641	59553.195	22975.227	88
8720.438	9293.938	59059.125	22777.734	87
8509.250	9080.020	57740.840	22497.262	85





8539.253	9383.323	57743.843	22497.262	85
8509.250	9080.020	57740.840	22497.262	85
8742.047	9296.078	59100.555	23193.941	87
8742.347	9296.378	59103.555	23193.941	87
8942.934	9516.383	60513.891	23963.277	89
8942.934	9516.333	60513.891	23963.277	89
8533.867	9091.070	57786.531	23202.270	85
8533.867	9091.070	57786.531	23202.270	85
8644.363	9187.563	58383.723	23618.335	86
8572.008	9108.578	57895.910	23416.492	85
8497.492	9076.617	57669.602	22820.629	85
8508.469	9118.977	57944.773	22886.316	85
8597.176	9188.266	58375.988	23122.918	86
8597.176	9188.266	58375.988	23122.918	86
8381.483	8974.480	57029.563	22600.805	84
8487.145	9083.973	57713.230	22960.879	85
8455.836	9031.363	57355.234	22955.418	85
8545.738	9116.273	57912.906	23104.480	85
8644.180	9189.793	58368.270	23380.313	86
8743.348	9258.473	58936.270	23676.867	87
8743.348	9258.473	58936.270	23676.867	87
8661.680	9177.152	58425.816	23420.723	86
8851.359	9408.347	59856.297	24497.145	88
8862.191	9407.402	59850.117	24589.336	88
8690.242	9216.586	58575.258	23538.727	85
8690.242	9216.586	58575.258	23538.727	85
8757.523	9322.008	59226.602	23803.586	86
8559.844	9092.683	57793.304	23335.932	84
8455.297	8968.395	57002.629	23070.516	83
8566.188	9097.359	57810.133	23451.563	84
8533.918	9051.233	57476.395	23373.191	84
8356.512	8853.906	56233.352	23111.988	82
8640.215	9158.590	58173.480	24051.336	85
8640.215	9158.590	58173.480	24051.336	85
8334.660	8856.813	56262.410	23159.477	81
8334.660	8856.813	56262.410	23159.477	81
8433.359	8971.641	56952.828	23447.852	82
8352.438	8873.773	56303.203	23456.773	81
8221.691	8727.199	55326.641	23408.347	80
8298.609	8822.453	55881.012	23749.559	81
8310.090	8813.516	55814.402	23766.363	81
7969.277	8483.932	53655.723	22744.230	77
7878.027	8359.711	52845.063	22499.031	76
7901.516	8363.168	52854.363	22525.199	76
7636.262	8089.211	51114.688	22221.828	74
7046.566	7448.090	46782.422	21097.824	69
7197.863	7639.227	47762.488	21327.523	71
7093.016	7486.508	46948.063	20977.824	70
7093.016	7486.508	46948.063	20977.824	70
7188.413	7575.633	47513.715	21203.328	71
7274.352	7651.285	47977.965	21261.223	72
7391.992	7784.285	48842.188	21599.258	73
7453.527	7852.160	49256.488	21777.934	74
7453.527	7852.160	49256.488	21777.934	74
7439.465	7802.323	48903.961	21476.523	74
7439.465	7802.323	48903.961	21476.523	74
7439.465	7802.323	48903.961	21476.523	74
7525.398	7874.215	49349.863	21568.831	75
7412.898	7785.066	48805.852	21243.102	74
7370.809	7736.492	48497.906	20742.430	74
7576.520	7977.758	50029.723	21061.203	76
7675.004	8093.523	50748.520	21463.156	77
7675.004	8093.523	50748.520	21463.156	77
7675.004	8093.523	50748.520	21463.156	77
7675.004	8093.523	50748.520	21463.156	77
7562.199	7973.983	49979.348	21115.938	76
7044.301	7387.645	46275.641	19767.758	72
6799.422	7167.336	44883.836	19402.430	70
6972.520	7373.547	46225.879	19879.980	72
6972.520	7373.547	46225.879	19879.980	72
6972.520	7373.547	46225.879	19879.980	72
6972.520	7373.547	46225.879	19879.980	72



6972.668	7362.828	46158.000	19825.605	72
6972.668	7362.828	46158.000	19825.605	72
6892.949	7313.727	45863.563	19426.836	71
6822.754	7227.508	45313.535	19276.730	70
6822.754	7227.508	45313.535	19276.730	70
6822.754	7227.508	45313.535	19276.730	70
6926.523	7327.459	45945.254	19601.637	71
7011.090	7417.492	46565.734	19741.484	71
7016.508	7434.137	46718.520	19553.102	71
6414.098	6760.402	42477.547	17514.895	66
6200.992	6539.828	41078.297	16802.613	64
6200.992	6539.828	41078.297	16802.613	64
6200.992	6539.828	41078.297	16802.613	64
6200.992	6539.828	41078.297	16802.613	64
6200.992	6539.828	41078.297	16802.613	64
6195.418	6536.668	41036.328	16851.863	64
6077.602	6417.133	40297.184	16481.809	63
6063.656	6382.641	40063.352	16295.809	63
6063.656	6382.641	40063.352	16295.809	63
6063.656	6382.641	40063.352	16295.809	63
6063.656	6382.641	40063.352	16295.809	63
5984.422	6322.426	39742.234	16078.887	62
6064.129	6407.837	40267.320	16448.336	63
5878.859	6194.145	38934.895	15919.223	61
5880.602	6195.109	38945.699	15795.293	61
5804.336	6117.270	38465.094	15500.125	60
5804.336	6117.270	38465.094	15500.125	60
5804.336	6117.270	38465.094	15500.125	60
5868.852	6181.328	38855.543	15645.359	61
5763.016	6056.063	38136.066	15102.938	60
5853.484	6143.520	38672.574	15217.984	61
5830.676	6136.578	38625.941	15237.840	61
5927.281	6221.152	39151.480	15308.988	62
5677.891	5954.227	37459.063	14555.051	59
5677.891	5954.227	37459.063	14555.051	59
5677.891	5954.227	37459.063	14555.051	59
5677.891	5954.227	37459.063	14555.051	59
5677.891	5954.227	37459.063	14555.051	59
5649.230	5933.844	37320.965	14482.594	59
5564.070	5826.348	36658.102	14346.613	58
5419.980	5666.895	35675.660	14039.016	57
5419.980	5666.895	35675.660	14039.016	57
5419.980	5666.895	35675.660	14039.016	57
5247.958	5500.430	34636.063	13823.656	55
5033.070	5301.797	33413.004	13493.215	53
5042.258	5282.191	33279.520	13469.785	53
5082.508	5315.191	33490.645	13512.969	53
4957.238	5186.398	32664.230	13189.230	52
4841.063	5066.895	31907.930	12872.953	51
4841.063	5066.895	31907.930	12872.953	51
4459.172	4655.566	29294.402	11931.258	47
4164.594	4319.980	27293.719	11010.559	44
4234.766	4392.910	27741.750	11366.188	45
4347.668	4507.879	28447.727	11720.230	46
4268.883	4413.395	27850.859	11524.199	45
4268.883	4413.395	27850.859	11524.199	45
4043.436	4181.134	26411.316	10712.605	42
4043.436	4181.134	26411.316	10712.605	42
3805.017	3967.632	25061.551	10213.688	40
3491.382	3616.181	22837.355	9251.688	37
3387.609	3516.219	22205.645	8926.785	36
3387.609	3516.219	22205.645	8926.785	36
3387.609	3516.219	22205.645	8926.785	36
3307.121	3437.268	21673.227	8647.273	35
3130.774	3221.222	20257.254	8710.902	33
2030.081	2069.101	12857.438	6041.777	23
2030.081	2069.101	12857.438	6041.777	23
2030.081	2069.101	12857.438	6041.777	23
2030.081	2069.101	12857.438	6041.777	23
2030.081	2069.101	12857.438	6041.777	23
2030.081	2069.101	12857.438	6041.777	23
2030.081	2069.101	12857.438	6041.777	23
2030.081	2069.101	12857.438	6041.777	23
2030.081	2069.101	12857.438	6041.777	23



2030.081	2069.101	12857.438	6041.777	23
2030.081	2069.101	12857.438	6041.777	23
2030.081	2069.101	12857.438	6041.777	23
2030.081	2069.101	12857.438	6041.777	23
1941.372	1975.810	12283.484	5650.359	22
1941.372	1877.727	11663.383	5351.777	21
1862.801	1764.397	10964.293	5087.355	20
1757.140	1764.397	10964.293	5087.355	20
1757.140	1764.397	10964.293	5087.355	20
1652.489	1633.699	10139.605	4851.383	19
1652.489	1633.699	10139.605	4851.383	19
1652.489	1633.699	10139.605	4851.383	19
1652.489	1633.699	10139.605	4851.383	19
1652.489	1633.699	10139.605	4851.383	19
1652.489	1633.699	10139.605	4851.383	19
1509.891	1495.909	9291.270	4651.305	17
1248.496	1250.705	7777.930	4107.512	14
1179.644	1189.099	7404.063	3833.480	13
893.271	885.982	5518.688	3187.829	10
893.271	885.982	5518.688	3187.829	10
893.271	885.982	5518.688	3187.829	10
893.271	885.982	5518.688	3187.829	10
893.271	885.982	5518.688	3187.829	10
719.582	709.953	4426.012	2465.060	8
652.915	634.553	3946.663	2132.944	7
652.915	634.553	3946.663	2132.944	7
485.505	456.022	2835.266	1642.962	5
485.505	456.022	2835.266	1642.962	5
485.505	456.022	2835.266	1642.962	5
393.570	377.377	2354.121	1431.575	4
393.570	377.377	2354.121	1431.575	4
393.570	377.377	2354.121	1431.575	4
393.570	377.377	2354.121	1431.575	4
234.332	227.682	1428.805	886.084	2
234.332	227.682	1428.805	886.084	2
234.332	227.682	1428.805	886.084	2
234.332	227.682	1428.805	886.084	2
234.332	227.682	1428.805	886.084	2
234.332	227.682	1428.805	886.084	2
121.429	112.714	722.821	532.036	1
121.429	112.714	722.821	532.036	1
121.429	112.714	722.821	532.036	1
121.429	112.714	722.821	532.036	1
121.429	112.714	722.821	532.036	1





# APPENDIX B

## DAILY AGGREGATED ENGINE PRODUCTION DATA (JULIAN DATE 0140 TO 1230)

LOAD NORM	DIRECT MAN HOURS	DIRECT LABOR COSTS	DIRECT MATERIAL COSTS	NUMBER IN SHOP
(APH)	(APL)	(APD)	(APM)	(N)
10.238	15.357	89.619	174.714	1
32.738	40.357	234.048	300.089	2
32.738	40.357	234.048	300.089	2
32.738	40.357	234.048	300.089	2
32.738	40.357	234.048	300.089	2
62.247	71.589	418.944	996.245	5
97.247	109.539	638.528	1161.911	6
116.998	132.750	773.690	1249.734	8
163.181	175.454	1019.103	1447.338	11
163.181	175.454	1019.103	1447.338	11
163.181	175.454	1019.103	1447.338	11
163.181	175.454	1019.103	1447.338	11
196.481	203.418	1180.281	1566.733	13
233.565	238.037	1381.204	1714.288	15
310.273	322.614	1873.376	2944.365	21
351.876	377.418	2188.176	3683.779	24
387.442	418.341	2427.248	4174.836	27
387.442	418.341	2427.248	4174.836	27
387.442	418.341	2427.248	4174.836	27
450.593	486.938	2827.821	4875.027	31
522.110	573.356	3327.181	5495.594	35
619.779	682.081	3957.847	6113.523	42
711.890	783.591	4550.105	6950.133	48
820.872	885.091	5139.813	8339.563	56
820.872	885.091	5139.813	8339.563	56
820.872	885.091	5139.813	8339.563	56
954.755	1014.835	5893.504	9906.590	65
1059.672	1134.940	6591.344	11038.004	72
1184.521	1262.447	7334.867	12581.531	81
1297.917	1374.551	8090.105	14448.273	90
1347.606	1439.495	8471.266	15115.770	94
1347.606	1439.495	8471.266	15115.770	94
1347.606	1439.495	8471.266	15115.770	94
1468.235	1553.490	9143.105	16078.527	103
1570.679	1645.712	9683.234	17328.438	109
1637.639	1719.075	10117.563	17906.961	115
1727.885	1830.884	10777.441	19188.352	121
1780.219	1885.048	11092.977	19715.563	125
1780.219	1885.048	11092.977	19715.563	125
1780.219	1885.048	11092.977	19715.563	125
1913.758	2008.894	11820.895	21042.152	134
1993.223	2109.874	12416.273	21866.098	140
1850.368	1952.694	11507.805	20519.855	132
1847.332	1956.487	11529.520	20831.133	132
1847.332	1956.487	11529.520	20831.133	132
1847.332	1956.487	11529.520	20831.133	132
1847.332	1956.487	11529.520	20831.133	132
1956.477	2044.057	12065.176	22547.125	137
2045.875	2136.082	12612.949	23466.750	144
2081.465	2170.875	12824.484	24441.367	146
2103.781	2172.144	12838.430	24507.488	148
2185.404	2218.432	13121.758	24923.074	154
2185.404	2218.432	13121.758	24923.074	154





2185.434	2218.432	13121.758	24923.074	154
2169.987	2225.660	13168.840	25009.359	151
2243.013	2290.976	13554.836	25969.266	155
2196.221	2281.349	13476.762	25330.895	153
2217.642	2307.885	13636.168	25932.395	156
2199.870	2291.459	13547.344	25993.504	156
2199.870	2291.459	13547.344	25993.504	156
2199.870	2291.459	13547.344	25993.504	156
2273.136	2356.841	13933.801	27068.387	162
2209.057	2310.741	13671.633	27041.293	155
2141.287	2241.099	13280.688	26222.727	154
2164.849	2265.998	13439.277	26746.586	156
2123.933	2221.423	13180.352	26905.391	156
2123.933	2221.423	13180.352	26905.391	156
2123.933	2221.423	13180.352	26905.391	156
2179.151	2288.755	13584.191	27258.195	159
2170.624	2291.256	13604.254	27184.559	159
2212.272	2332.827	13854.398	27300.375	160
2243.109	2347.899	13948.133	27569.320	162
2053.735	2156.384	12745.852	25083.586	149
2053.735	2156.384	12745.852	25083.586	149
2053.735	2156.384	12745.852	25083.586	149
2141.597	2239.777	13247.105	25977.988	154
2193.203	2289.333	13561.125	26184.512	156
2223.891	2318.945	13744.941	26252.277	157
2273.914	2369.616	14056.105	26685.652	158
2253.458	2355.984	13979.188	26557.063	157
2253.458	2355.984	13979.188	26557.063	157
2253.458	2355.984	13979.188	26557.063	157
2272.812	2366.238	14057.887	25877.008	158
2279.969	2372.140	14108.426	25407.242	157
2337.473	2422.016	14419.531	25812.938	160
2273.998	2361.695	14063.449	25305.254	156
2293.521	2392.597	14248.477	24827.348	155
2293.521	2392.597	14248.477	24827.348	155
2293.521	2392.597	14248.477	24827.348	155
2227.643	2315.933	13805.656	23838.453	150
2214.127	2280.857	13598.379	23757.438	146
2352.957	2408.003	14362.227	25030.324	154
2377.771	2442.025	14572.656	25509.402	157
2385.736	2443.906	14584.469	25569.352	156
2385.736	2443.906	14584.469	25569.352	156
2385.736	2443.906	14584.469	25569.352	156
2416.795	2473.775	14790.313	26020.281	156
2443.079	2490.207	14903.352	26398.730	157
2425.910	2463.983	14755.770	25781.961	155
2463.172	2492.698	14929.508	26146.199	157
2384.907	2411.459	14438.918	25212.953	149
2384.907	2411.459	14438.918	25212.953	149
2384.907	2411.459	14438.918	25212.953	149
2322.838	2338.083	13990.992	24568.703	143
2272.734	2284.012	13673.430	24090.977	139
2218.145	2248.787	13461.012	23237.008	134
2240.199	2283.055	13663.703	23280.688	136
2259.255	2297.550	13739.395	23504.168	138
2259.255	2297.550	13739.395	23504.168	138
2259.255	2297.550	13739.395	23504.168	138
2259.255	2297.550	13739.395	23504.168	138
2208.100	2250.093	13438.461	23149.910	135
2112.230	2157.892	12868.273	21848.625	131
2017.763	2078.118	12357.859	21013.645	126
1952.255	2009.323	11880.723	20358.461	120
1952.255	2009.323	11880.723	20358.461	120
1952.255	2009.323	11880.723	20358.461	120
1975.915	2036.432	12028.977	20540.617	120
2059.602	2107.962	12421.313	21180.117	125
1979.552	2028.811	11948.645	20340.715	120
1957.291	2011.630	11836.348	19963.445	121
1926.598	1974.861	11611.828	19454.895	120
1926.598	1974.861	11611.828	19454.895	120
1926.598	1974.861	11611.828	19454.895	120
1916.426	1955.240	11481.965	19366.641	117



1838.461	1879.018	10990.992	19181.051	112
1793.401	1841.287	10750.059	18568.984	113
1854.512	1898.213	11083.027	19526.848	118
1754.202	1768.254	10299.570	18082.520	113
1754.202	1768.254	10299.570	18082.520	113
1754.202	1768.254	10299.570	18082.520	113
1777.624	1788.357	10426.902	18116.066	119
1954.078	1909.218	11130.500	19242.391	126
2061.090	2015.235	11757.211	20384.395	133
1895.039	1856.838	10824.781	18465.984	127
1919.537	1925.025	11239.008	19087.203	133
1919.537	1925.025	11239.008	19087.203	133
1919.537	1925.025	11239.008	19087.203	133
1927.038	1936.551	11307.762	19036.301	133
2047.790	2047.170	11959.719	20148.270	140
2038.441	2024.141	11831.762	20060.016	139
2064.855	2038.333	11918.301	20585.258	139
2099.739	2087.302	12213.363	20585.520	142
2099.739	2087.302	12213.363	20585.520	142
2099.739	2087.302	12213.363	20585.520	142
2149.584	2133.614	12493.684	21070.352	145
2149.990	2114.021	12370.492	21302.902	147
2214.447	2179.903	12759.563	21669.297	149
2244.454	2216.655	12984.293	21853.391	151
2251.182	2217.386	12998.609	21533.672	152
2251.182	2217.386	12998.609	21533.672	152
2251.182	2217.386	12998.609	21533.672	152
2217.045	2169.516	12724.938	21447.500	148
2270.033	2243.110	13158.668	22143.836	153
2272.864	2241.917	13144.660	22318.293	150
2393.319	2357.688	13834.074	23479.266	155
2442.375	2394.245	14053.609	23607.379	158
2442.375	2394.245	14053.609	23607.379	158
2442.375	2394.245	14053.609	23607.379	158
2400.364	2341.960	13744.172	23088.195	156
2409.148	2351.777	13806.594	23075.355	155
2489.552	2423.009	14226.023	23752.688	160
2578.118	2516.648	14780.023	24204.836	165
2418.144	2379.404	13984.313	22404.117	161
2418.144	2379.404	13984.313	22404.117	161
2424.305	2389.677	14047.477	22444.129	162
2358.554	2334.954	13728.438	21634.477	157
2363.691	2342.388	13780.594	21736.129	158
2470.512	2417.843	14234.691	21909.078	161
2532.558	2465.694	14519.363	22702.758	164
2392.987	2301.969	13569.656	21451.008	156
2392.987	2301.969	13569.656	21451.008	156
2392.987	2301.969	13569.656	21451.008	156
2311.929	2259.940	13340.020	21299.508	156
2226.540	2185.640	12921.816	20637.934	153
2226.540	2185.640	12921.816	20637.934	153
2329.938	2250.094	13315.801	21196.734	157
2377.325	2292.502	13596.520	21804.676	158
2377.325	2292.502	13596.520	21804.676	158
2384.633	2301.156	13648.398	21875.250	159
2391.472	2317.274	13775.242	22064.793	158
2377.097	2286.254	13617.402	22127.516	157
2296.971	2203.856	13173.133	21742.043	154
2282.368	2195.536	13162.203	21930.922	153
2310.688	2219.738	13327.535	22186.242	155
2310.688	2219.738	13327.535	22186.242	155
2310.688	2219.738	13327.535	22186.242	155
2255.581	2171.949	13087.820	21971.555	154
2206.175	2166.062	13087.234	21620.309	152
2255.091	2219.743	13504.633	21854.613	155
2255.091	2219.743	13504.633	21854.613	155
2255.091	2219.743	13504.633	21854.613	155
2255.091	2219.743	13504.633	21854.613	155
2255.091	2219.743	13504.633	21854.613	155
2291.948	2242.715	13706.863	21982.742	157
2291.948	2242.715	13706.863	21982.742	157
2304.609	2248.565	13751.102	22114.520	156





2399.029	2368.983	14522.301	23193.070	161
2321.938	2303.178	14158.152	22246.195	157
2292.934	2285.167	14072.035	22105.320	153
2292.934	2285.157	14072.035	22105.320	153
2292.934	2285.157	14072.035	22105.320	153
2321.181	2314.662	14253.602	22698.801	153
2325.064	2327.428	14336.516	22779.004	152
2233.604	2244.033	13844.117	21980.891	146
2156.739	2177.388	13437.500	21257.938	140
1909.044	1963.221	12156.793	18933.691	126
1893.403	1948.041	12060.465	18862.387	125
1893.403	1948.041	12060.465	18862.387	125
1832.633	1894.000	11748.105	18715.652	123
1858.779	1908.614	11838.719	18549.523	124
1815.513	1923.148	11959.930	17814.254	121
1784.839	1897.668	11788.379	17195.148	118
1567.987	1676.026	10414.949	14318.551	105
1567.987	1676.026	10414.949	14318.551	105
1589.968	1697.130	10545.156	14474.352	108
1607.181	1718.125	10617.949	14941.840	107
1502.356	1591.602	9729.816	14307.145	106
1400.122	1501.758	9143.383	13812.891	102
1400.122	1501.758	9143.383	13812.891	102
1400.122	1501.758	9143.383	13812.891	102
1400.122	1501.758	9143.383	13812.891	102
1479.642	1583.352	9649.020	14583.203	106
1582.099	1687.594	10291.676	15411.609	110
1723.388	1809.670	11045.543	16586.688	118
1768.666	1851.795	11303.742	16872.188	123
1768.666	1851.795	11303.742	16872.188	123
1768.666	1851.795	11303.742	16872.188	123
1768.666	1851.795	11303.742	16872.188	123
1763.229	1830.750	11166.953	16520.922	117
1879.084	1951.352	11908.578	17849.500	124
1957.151	2002.179	12220.738	18439.813	129
1947.919	1997.444	12203.250	18505.547	128
1995.507	2056.234	12573.695	19288.098	135
1995.507	2056.234	12573.695	19288.098	135
1995.507	2056.234	12573.695	19288.098	135
1992.799	2053.705	12570.160	19681.461	134
2071.122	2110.258	12932.793	20343.715	138
2150.554	2180.157	13368.520	21345.293	142
2123.549	2146.904	13170.418	21020.727	140
2191.584	2218.776	13620.527	21745.410	148
2191.584	2218.776	13620.527	21745.410	148
2191.584	2218.776	13620.527	21745.410	148
2190.433	2209.055	13567.063	22616.961	144
2237.346	2231.360	13706.703	23117.801	144
2278.872	2259.077	13879.926	24311.688	145
2297.144	2284.130	14038.824	24117.824	147
2260.396	2268.922	13958.906	23878.387	145
2260.396	2268.922	13958.906	23878.387	145
2260.396	2268.922	13958.906	23878.387	145
2259.584	2257.143	13896.152	23851.090	144
2243.095	2235.063	13814.473	23840.223	144
2209.112	2180.721	13486.348	23516.277	141
2250.747	2206.899	13651.512	24033.102	142
2217.262	2157.448	13344.980	23873.977	138
2217.262	2157.448	13344.980	23873.977	138
2217.262	2157.448	13344.980	23873.977	138
2192.121	2126.508	13153.074	24098.359	135
2200.390	2140.052	13241.984	24087.879	136
2187.983	2145.963	13292.070	23812.250	136
2236.242	2196.978	13610.098	24323.809	139
2174.818	2119.185	13135.824	23540.684	135
2174.818	2119.185	13135.824	23540.684	135
2174.818	2119.185	13135.824	23540.684	135
2092.309	2040.400	12660.625	22568.742	126
2076.020	2028.319	12602.402	22274.477	125
2134.898	2082.856	12945.699	22736.719	128
2171.604	2111.118	13158.344	23124.145	131
2063.534	2013.736	12563.551	22430.613	124



2063.534	2013.736	12563.551	22430.613	124
2063.534	2013.736	12563.551	22430.613	124
2063.534	2013.736	12563.551	22430.613	124
2114.981	2045.041	12781.516	23078.625	124
2147.295	2085.041	13051.223	23508.215	126
2147.481	2100.880	13165.895	23818.828	125
2040.191	2001.864	12551.695	21498.934	118
2040.191	2001.864	12551.695	21498.934	118
2040.191	2001.864	12551.695	21498.934	118
2035.035	1999.333	12547.648	20392.738	118
1950.622	1920.179	12078.543	19560.930	114
1964.544	1936.628	12174.855	20164.738	115
1941.610	1934.983	12182.301	19780.031	115
1925.311	1932.916	12155.473	19796.434	114
1925.311	1932.916	12155.473	19796.434	114
1925.311	1932.916	12155.473	19796.434	114
1935.334	1947.720	12257.215	20164.801	115
1891.085	1865.521	11740.680	20292.184	110
1919.283	1869.695	11759.840	20646.965	110
1985.071	1907.403	11991.688	21165.242	115
2015.649	1937.262	12145.848	21628.348	116
1998.982	1919.762	12036.262	21411.559	115
1998.982	1919.762	12036.262	21411.559	115
2084.852	1979.994	12397.848	22773.602	118
2158.962	2046.664	12799.359	23237.594	121
2165.525	2061.844	12879.867	23217.383	121
2205.617	2102.592	13131.426	23439.566	124
2000.034	1924.109	12006.781	21866.645	113
2000.034	1924.109	12006.781	21866.645	113
1833.646	1829.419	11385.977	20920.484	106
1889.502	1901.983	11812.801	21969.965	111
1834.897	1848.562	11466.734	20691.352	108
1777.131	1848.278	11422.117	20226.266	106
1577.503	1643.498	10078.977	18372.465	99
1577.563	1643.498	10078.977	18372.465	99
1577.563	1643.498	10078.977	18372.465	99
1609.657	1664.423	10181.387	19589.184	101
1579.952	1619.134	9860.684	19565.410	97
1604.964	1643.502	9977.668	19705.188	97
1610.700	1647.824	9967.617	19374.480	96
1663.756	1692.639	10238.844	19799.035	100
1663.756	1692.639	10238.844	19799.035	100
1663.756	1692.639	10238.844	19799.035	100
1672.789	1703.284	10300.680	19889.262	101
1863.411	1873.577	11321.266	22260.297	110
1937.092	1953.310	11800.535	23048.438	114
1653.943	1662.032	10044.113	19199.145	92
1623.999	1650.436	9971.188	18914.961	90
1623.999	1650.436	9971.188	18914.961	90
1623.999	1650.436	9971.188	18914.961	90
1617.437	1631.214	9845.828	19024.805	90
1675.752	1682.365	10153.648	19729.195	92
1748.159	1742.433	10508.977	20337.270	93
1814.956	1791.956	10809.910	20804.629	98
1801.028	1771.113	10686.352	20560.230	99
1801.028	1771.113	10686.352	20560.230	99
1801.028	1771.113	10686.352	20560.230	99
1801.810	1780.688	10740.477	20412.352	96
1884.122	1858.425	11206.469	21844.129	102
1955.906	1916.394	11556.992	22178.629	104
1992.589	1944.860	11739.477	22693.184	106
1999.899	1948.669	11763.078	22590.785	107
1999.899	1948.669	11763.078	22590.785	107
1999.899	1948.669	11763.078	22590.785	107
2041.375	1961.091	11845.012	22893.238	106
2047.644	1968.615	11894.078	23324.730	107
2060.607	1968.070	11903.559	23543.828	107
2047.332	1944.550	11769.793	23377.469	107
2093.233	1987.372	12034.617	23868.855	109
2093.233	1987.372	12034.617	23868.855	109
2093.233	1987.372	12034.617	23868.855	109





2089.667	1982.586	11992.305	22894.574	107
2145.323	2036.544	12355.063	23878.516	111
2103.922	1990.918	12097.965	23387.523	109
2133.990	2026.579	12328.309	23581.070	113
2144.836	2024.525	12323.465	23180.527	111
2144.836	2024.525	12323.465	23180.527	111
2090.576	1976.109	12037.230	22187.113	109
2127.495	2021.610	12339.043	22545.770	109
2153.834	2052.146	12541.547	22991.344	113
2160.065	2047.016	12525.320	22593.652	114
2043.272	1950.883	11920.422	21124.578	107
2061.014	1965.818	12012.000	21265.547	108
2061.014	1965.818	12012.000	21265.547	108
2088.276	1997.365	12237.574	21906.672	107
2075.321	1968.170	12088.398	22095.465	107
2088.463	1998.655	12283.148	22197.781	108
2162.171	2070.580	12754.211	23536.094	113
2219.256	2123.733	13096.883	23843.602	115
2219.256	2123.733	13096.883	23843.602	115
2219.256	2123.733	13096.883	23843.602	115
2208.469	2099.390	12972.004	23573.988	117
2147.177	2057.032	12744.277	23039.180	112
2104.275	2008.271	12487.520	22826.754	109
2137.983	2032.352	12672.359	23175.809	110
2155.364	2046.816	12765.008	23648.742	113
2155.364	2046.816	12765.008	23648.742	113
2155.364	2046.816	12765.008	23648.742	113
2121.387	2020.873	12626.367	22965.152	110
2096.097	2013.960	12623.898	22817.184	111
2080.096	2021.276	12688.602	22986.375	110
2056.515	1997.908	12550.754	22694.117	109
2084.364	2016.952	12670.117	23186.598	111
2084.364	2016.952	12670.117	23186.598	111
2084.364	2016.952	12670.117	23186.598	111
2119.373	2048.469	12866.078	23928.895	111
2132.973	2055.002	12906.359	24166.918	110
2130.551	2053.768	12900.738	24051.844	108
2086.451	2018.356	12680.383	23567.063	103
2086.451	2018.356	12680.383	23567.063	103
2086.451	2018.356	12680.383	23567.063	103
2107.167	2031.945	12752.879	23970.813	105
2150.157	2063.155	12970.297	24463.688	108
2125.182	2038.965	12796.852	24035.395	106
2105.568	2008.626	12618.051	23986.852	105
2045.870	1955.613	12282.531	23489.211	102
2045.870	1955.613	12282.531	23489.211	102
2045.870	1955.613	12282.531	23489.211	102
1891.854	1834.740	11497.520	21344.070	92
1843.795	1782.694	11148.777	20913.266	89
1754.614	1691.659	10568.688	19729.094	85
1679.812	1621.631	10077.922	19141.859	81
1679.812	1621.631	10077.922	19141.859	81
1679.812	1621.631	10077.922	19141.859	81
1647.593	1606.623	9951.102	18721.465	78
1533.660	1504.470	9269.934	17216.988	73
1515.147	1465.930	9114.781	17858.117	71
1451.174	1407.961	8663.602	17830.387	70
1550.540	1506.806	9265.586	19657.926	77
1550.540	1506.806	9265.586	19657.926	77
1550.540	1506.806	9265.586	19657.926	77
1631.855	1584.807	9742.348	20835.359	81
1753.377	1693.912	10405.617	22760.770	88
1871.918	1812.327	11131.852	24873.453	94
1735.045	1685.274	10359.902	22862.020	86
1545.239	1505.248	9257.008	20933.070	77
1548.643	1510.886	9290.922	20961.578	78
1548.643	1510.886	9290.922	20961.578	78
1548.643	1510.886	9290.922	20961.578	78
1464.633	1426.490	8777.520	19978.367	75



[illegible]



# APPENDIX C

## MONTHLY AGGREGATED COMPONENT PRODUCTION DATA (JULY 70 TO SEPTEMBER 71)

LOAD NORM	DIRECT MAN HOURS	DIRECT MATERIAL COSTS
(MPH)	(MPL)	(MPM)
120192.8	131572.0	701456.44
127197.4	155765.8	1194907.00
206383.3	149168.2	1262821.00
149881.1	162116.4	1040680.19
128056.5	132349.0	1030891.25
138919.1	113285.3	1114578.00
103123.8	133633.8	1183543.00
151701.8	138997.7	1299405.00
176070.3	164260.8	1383226.00
154349.9	154349.9	1572007.00
130690.0	135170.7	900192.63
161695.1	166237.1	1523931.00
144559.1	144559.1	777628.69
138600.0	150480.0	1255478.00
180592.4	167887.5	1246541.00



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KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Cobb-Douglas Production Functions						
Production Function						
Naval Air Rework Facility						
Regression Analysis						









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